Exercise 1  Homogeneous Coordinates  (7 Points)

a) Verify if the following points $b_1 = (1, -1, 1)$ and $b_2 = (1, 1, 1)$ are located on the line $x = (-1, 1, -1)$; (2 Points)

b) Explain what is a line at infinity. Can you mention at least two of its properties? (3 Points)

c) Three planes through the origin are intersecting with one another in an angle of 45 degrees. When they intersect with the projecting plane what geometric figure do they produce? (1 Points)

d) Show that the conic section $x_1^2 + x_1x_2 + x_2^2 - x_1x_3 - x_2x_3 - x_3^2 = 0$ is the same as the conic section with $C = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$ (Hint: do you know how to multiply matrices?) (4 Points)

Exercise 2  Projective transformations  (10 Points)

a) Show that $R^T R = I$ where $R$ is the rotation matrix. Explain all your steps. (Hint: do you know how to transpose matrices?) (4 Points)

b) Given the line $x = (1, 3, 1)$, you want to rotate it of an angle $\alpha = \pi$ and transpose it of 2 both in $x_1$ and $x_2$ direction. Compute the new line equation. (6 Points)