

Übungsblatt 10

Abgabe: Dienstag, 10.07.07 bis 13 Uhr

Exercise 1 (Unification, Substitution, Resolution) (40 Points)

a) [5 pts.] For each pair of atomic sentences below, check if they can be unified. If so, specify the most general unifier and the resulting formula. If not, why?

1. $P(A,B,B), P(x,y,z)$
2. $P(f(y),w, g(z, y)), P(x, x, g(z,A))$
3. $Q(y, g(A,B)), Q(g(x,x), y)$
4. $Older(Father(y), y), Older(Father(x), John)$
5. $Knows(Father(y), y), Knows(x, x)$

b) [15 pts.] From “Horses are animals,” it follows that “The head of a horse is the head of an animal.” Demonstrate that this inference is valid by carrying out the following steps:

1. Translate the premise and the conclusion into FOL. Use three predicates: $HeadOf(h,x)$ (meaning “h is the head of x”), $Horse(x)$, and $Animal(x)$.
2. Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.
3. Use resolution to show that the conclusion follows from the premise.

c) [20 pts.] Given the following sentences in FOL:

(A) $\forall x \exists y (x \geq y)$

(B) $\exists y \forall x (x \geq y)$

1. Assume that the variables range over all the natural numbers $0,1,2,\dots,\infty$ and that the “ \geq ” predicate means “is greater than or equal to.” Under this interpretation, translate (A) and (B) into English (or German).
2. Is (A) true under this interpretation?
3. Is (B) true under this interpretation?

6. Using resolution, try to prove that $(B) \models (A)$. Proceed with the proof even if you think that $(B) \not\models (A)$; continue until the proof fails (if this it fails). Show the unifying substitution for each resolution step. If the proof fails, explain why it failed.

7. Now try to prove that $(A) \models (B)$