Exercise Sheet WS 14/15
Wirtschaftsinformatik und Maschinelles Lernen (ISMLL)
Ruth Janning, M.Sc., Carlotta Schatten M.Eng.

Exercise Sheet 6

Submission: Tuesday, 13.01.2014, 10:00

Exercise 1 Propositional Logic (20 Points)

a) Write the truth tables of :

- $\neg A \wedge B$
- $\neg A \Rightarrow B$
- $(\neg A) \vee A$
- $A \wedge (B \vee C)$
- $A \vee (B \wedge \neg C)$
- $A \Rightarrow (\neg B \wedge C)$

(6 Points)

b) Prove with transformations following equivalences and mention at each step what rule (commutation, association, De Morgan, etc.) you apply.

- $\neg\alpha \wedge \beta \equiv \neg(\beta \vee \alpha)$
- $(\alpha \wedge \neg\beta \vee \gamma) \equiv (\gamma \vee \alpha) \wedge (\gamma \vee \neg\beta)$
- $\alpha \Leftrightarrow \beta \equiv (\neg\alpha \wedge \neg\beta) \vee (\beta \wedge \alpha)$

(6 Points)

c) Demonstrate which characteristics (tautology, contradiction, satisfiable) have this formulas with transformations or truth tables. Explain all your steps.

(a) $\neg((A \Rightarrow (B \Rightarrow C)) \vee (A \vee B \vee \neg C))$
(b) $\neg(((A \wedge \neg B) \vee C) \vee ((A \Rightarrow B) \Rightarrow C))$

(4 Points)

d) Prove $\boldsymbol{G}$ from $\mathcal{F}$ using the proof method suggested by the scripts.
$\boldsymbol{G} := (is\_penguin \wedge cannot\_fly)$
$\mathcal{F} := \{has\_feathers,\ walks\_only,$
$(is\_bird \wedge cannot\_fly) \Rightarrow is\_penguin,$
$has\_feathers \Rightarrow is\_bird,$
$(is\_bird \wedge walks\_only) \Rightarrow cannot\_fly\}$
Consider that each derivation can be used with $\mathcal{F}$ ones for new derivations.

Basic rules for derivation

premise	conclusion	name
$G \in \mathcal{F}$	$\mathcal{F} \vdash G$	assumption
$\mathcal{F} \vdash G, \mathcal{F} \subseteq \mathcal{F}'$	$\mathcal{F}' \vdash G$	monotonicity
$\mathcal{F} \vdash \neg\neg G$	$\mathcal{F} \vdash G$	double negation
$\mathcal{F} \vdash F, \mathcal{F} \vdash G$	$\mathcal{F} \vdash F \wedge G$	$\wedge$-introduction
$\mathcal{F} \vdash F \wedge G$	$\mathcal{F} \vdash F$	$\wedge$-elimination
$\mathcal{F} \vdash F \wedge G$	$\mathcal{F} \vdash G \wedge F$	$\wedge$-symmetry
$\mathcal{F} \vdash F$	$\mathcal{F} \vdash F \vee G$	$\vee$-introduction
$\mathcal{F} \vdash F \vee G$, $\mathcal{F} \cup \{F\} \vdash H, \mathcal{F} \cup \{G\} \vdash H$	$\mathcal{F} \vdash H$	$\vee$-elimination
$\mathcal{F} \vdash F \vee G$	$\mathcal{F} \vdash G \vee F$	$\vee$-symmetry
$\mathcal{F} \cup \{F\} \vdash G$	$\mathcal{F} \vdash F \rightarrow G$	$\rightarrow$ -introduction
$\mathcal{F} \vdash F, \mathcal{F} \vdash F \rightarrow G$	$\mathcal{F} \vdash G$	$\rightarrow$ -elimination
$\mathcal{F} \vdash F$	$\mathcal{F} \vdash (F)$	$()$-introduction
$\mathcal{F} \vdash (F)$	$\mathcal{F} \vdash F$	$()$-elimination
$\mathcal{F} \vdash ((F \wedge G) \wedge H)$	$\mathcal{F} \vdash F \wedge G \wedge H$	$\wedge$-parentheses rule
$\mathcal{F} \vdash ((F \vee G) \vee H)$	$\mathcal{F} \vdash F \vee G \vee H$	$\vee$-parentheses rule

(4 Points)