Business Analytics
Exercise Sheet 6

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Exercise 17: Regularized Polynomial Regression (5 points)

A **regularized** polynomial of degree three captures interaction among (p-many) predictors characterized by the coefficients of the monomials, as depicted in Equation 1. Here we assumed the regularization parameter $\lambda = 1$.

$$\begin{aligned} \hat{y}(x,\beta) &:= \beta_0 + \sum_{i=1}^{p} \beta_i x_i + \sum_{i=1}^{p}\sum_{j=1}^{p} \beta_{i,j} x_i x_j + \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} \beta_{i,j,k} x_i x_j x_k + \text{Reg}(\beta) \qquad (1) \\ \text{Reg}(\beta) &:= \beta_0^2 + \sum_{i=1}^{p} \beta_i^2 + \sum_{i=1}^{p}\sum_{j=1}^{p} \beta_{i,j}^2 + \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} \beta_{i,j,k}^2 \end{aligned}$$

(a) What is overfitting? Describe the positive effect of the regularization term in order to avoid overfitting?

(b) Learn the coefficients of a polynomial regression of degree three using gradient descent, aiming to optimize for the L2 loss for the toy dataset of Table 1. Please apply five iterations of the gradient descent technique using the learning rate value $\eta = 0.1$ and show the intermediate values of the coefficients β after each iteration. Initially all the coefficient values are zero.

Tabelle 1: Toy Regression Dataset

$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{y}$
2	1	1
2	0	2
3	1	2
3	0	3
4	1	3

(c) How does the L2-loss varies from one iteration to the other. Do you see a convergence?

Exercise 18: Closed-form Polynomial Regression (5 points)

(a) How can a ridge regression problem be solved as a system of linear equations? Provide the closed-form solution of β.

(b) How can the closed-form solution of part a) be used to solve the coefficients (β) of the polynomial regression in Exercise 17? Formulate your solution clearly.

(c) Provide your opinions on the advantages and disadvantages of the closed-form solution compared to the gradient descent technique.

Submission

- Electronically to wistuba@ismll.de. Text submitted as pdf, code submitted as source files. No archives.