Big Data Analytics
Exercise Sheet 2

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May 8th, 2015
Submission until May 15th, 2015, to khan@ismll.de

Exercise 1: Gradient Descent (5 Points)

a) Apply gradient descent on the function $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{2}x^2$ with following configurations:

a) Use step length $\alpha = 0.3$ and starting point $x_0 = -1$ and show the first four iterations. What is your minimum?

b) Use step length $\alpha = 2$ and starting point $x_0 = -1$ and show the first four iterations. What has happened and why?

c) Use step length $\alpha = 0.3$ and starting point $x_0 = 0$ and show the first two iterations. What has happened and why?
Do the same again with $\alpha = 0.8$ and starting point $x_0 = 0.5$ and show the first four iterations. Where is your minimum now?
What would be a possible solution to overcome the problem just identified?

Exercise 2: Linear Regression using Gradient Descent(10 Points)

A restaurant chain wants to predict profit given the population of the city, in which they want to open a new franchise. Attached is the restaurant data set with two columns, Population of the city and second column is Profit earned in that city.

a) To visualize this data, you are required to plot this data by putting "Profits in 10,000s" as label of Profit on y-axis and "Population of city in 10,000s" as label of Population on x-axis.

b Program a Gradient Descent algorithm that minimizes cost function of Uni-variate Linear Regression by learning optimal values for 'β'. The objective of linear regression is to minimize the cost function

$$J(\beta) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\beta(x^i) - y^i\right)^2$$

Where the hypothesis $h_\beta(x)$ is given by linear model

$$h_\beta(x) = \beta^T x = \beta_0 + \beta_1 x$$

Exercise 3: Logistic Regression (5 Points)

The table below presents the test-firing results for 25 surface-to-air anti aircraft missiles at targets of varying speed. The result of each test is either a hit (t = 1) or a miss (t = 0). The explanatory variable x gives the speed of the target in knots.

Test	Target speed (x) in knots	t	Test	Target speed (x) in knots	t
1	400	0	14	330	1
2	220	1	15	280	1
3	490	0	16	210	1
4	210	1	17	300	1
5	500	0	18	470	1
6	270	0	19	230	0
7	200	1	20	430	0
8	470	0	21	460	0
9	480	0	22	220	1
10	310	1	23	250	1
11	240	1	24	200	1
12	490	0	25	390	0
13	420	0			

Logistic Regression model is given by:

$$\hat{y}(\mathbf{x};\beta) = p(t=1|x) = \frac{exp(\beta_0 + \beta_1 x)}{1 + exp(\beta_0 + \beta_1 x)}$$

Using maximum likelihood, we estimated the following values of parameters.
$\beta_0 = 6.071$ and $\beta_1 = -0.018$

a) β_1 is negative. What can we infer from that about the relation between target speed and the probability of a hit?

b) What is the fitted probability of a hit for a target speed of 350 knots?

c) Give the classification rule corresponding to the fitted model, assuming we always allocate to the class with highest fitted probability for a given target speed.