Tutorial Bayesian Network WS 2007/08
Wirtschaftsinformatik und Maschinelles Lernen (ISMLL)
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Tutorial 3

Basics of Graph Theory

Solutions should be given till 12th November 2007, 16:00

Exercise 1 u-separation (10 Points)

a) [5 pts.] The graph in Figure 1 is given. What is the smallest set Z (i.e. the set with minimal amount of vertexes) so that {A} and {B} are u-separated?
b) [5 pts.] The graph in Figure 2 is given. Are the sets of nodes {A,B} and {F,G,H} u-separated by Z ?
   a. Z={C,D}
   b. Z={C,D,E}

Exercise 2 Blocking (10 Points)

a) [2 pts.] Consider the graph in Figure 3. Which are the ancestors and descendents of {J,I,D}, {L,F} and {L,M} ?
b) [5 pts.] At which positions is the chain A,B,C,D,E,F,G,H blocked by {I,D,J}, {L,F} and {L,M}
c) [3 pts.] Is it possible to choose directions for all edges of the graph in Figure 1, so that the empty set does not block any node of any chain?

Exercise 3 d-separation (10 Points)

a) [3 pts.] Consider the graph in Figure 4. Construct the moral graph of this graph!
b) [3 pts.] Are the sets {A,B,C,D,E} and {I,J,K,L} d-separated by {F,G,H} in the original graph? (Please answer this question based on the moral graph.)
c) [4 pts.] Are the sets {A,B,C,D,E} and {I,J,K,L} d-separated by {F,G,H} in the original graph? (Please answer this question based on the "first" definition of d-separation.)

Bonus Exercise (optional) Paths (15 Points)

a) [2 pts.] Consider the graph in Figure 1. We say, two paths are disjoint, if the corresponding sequences do not have any common edge. For example (E,H,G,C,A,D) and (E,F,I,D) are two disjoint paths. What is the amount of disjoint paths between A and B? We do not allow cycles in the path (the same vertex should only be used once.)
b) [8 pts.] Two paths are said to be different if their sequences are not the same. For example the paths {A, C, B} and {A, C, G, I, F, B} are different. (They have common elements, but the whole sequences are not the same.) What is the amount of different paths between A and B? We do not allow cycles in the path (the same vertex should only be used once.) (Please explain your answer in details.)
c) [5 pts.] Implement a computer program, which solves the previous task.

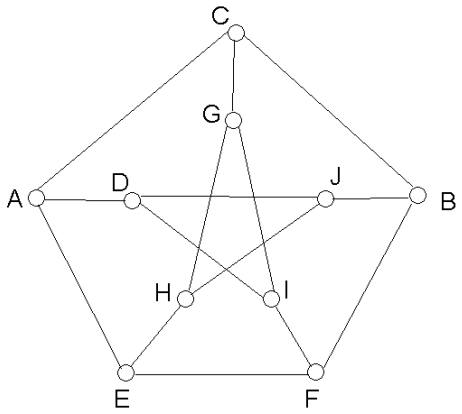

Fig.1 Petersen Graph

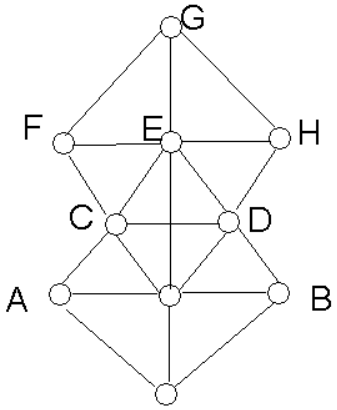

Fig. 2

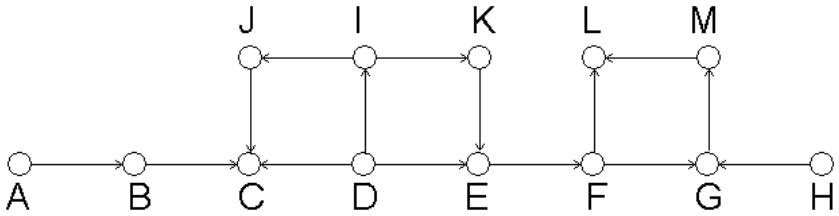

Fig. 3

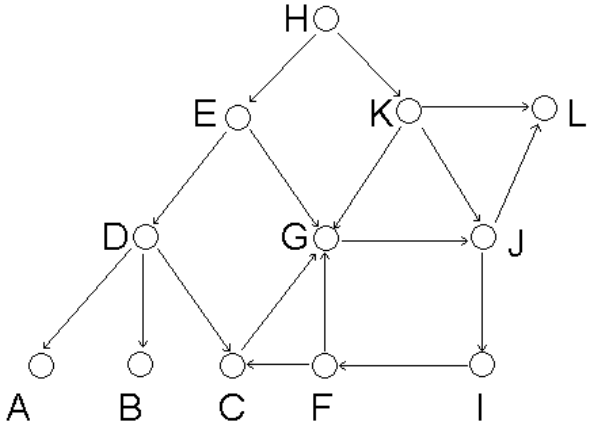

Fig. 4