

Tutorial 4

Markov Networks (1.)

Solutions should be given till 19th November 2007, 16:00

Exercise 1 Topological order (5 points + 10 bonus points)

- [3 pts.] Does the graph in Figure 1 have a topological order? If yes, is this unique? (Does it have only one topological order or more?)
- [2 pts.] Show an example for a graph, which has several topological orders!
- [optional, 10 bonus pts.] Is it possible to choose directions for the edges of the Petersen graph (Fig. 2.) so that it has a unique topological order?

Exercise 2 Graphical representation of independence (15 Points)

- [4 pts.] Is the graph in Fig. 3 a representation of the following independence model?
 $I = \{I(A,C|\{E,F,B\}), I(C,A|\{E,F,B\}), I(A,C|\{D,F,B\}), I(C,A|\{D,F,B\}), I(A,D|\{E,F,B\}), I(D,A|\{E,F,B\}), I(A,D|\{E,F,C\}), I(D,A|\{E,F,C\})\}$
- [3 pts.] Is the graph in Fig. 3 a faithful representation of the indep. relation above?
- [3 pts.] Construct (another) graph, which represents the independence relation above! (This graph should not necessary be a faithful representation.)
- [5 pts.] For which independence relation is the graph in Fig. 3 a faithful representation?

Exercise 3 Properties of independency models, graphical representation (10 Points)

Suppose we are given the following independence model:

$$I(A,B|\{C,D\}), I(B,A|\{C,D\}), I(A,C|\{D,E\})$$

- [3 pts.] Which of the properties (symmetric, decomposable and intersectable) hold for this model?
- [2 pts.] Modify the model so that all of the properties above hold for the model!
- [5 pts.] Construct the minimal undirected graph representation of the model. Is it trivial? Is it unique? Is it faithful?

Exercise 4 Potentials, Markov Networks (15 Points)

Given that the potentials ψ_1, ψ_2 represented in Table 1 and Table 2 factorize the JPD p , solve the following tasks:

- [4 pts.] Multiply ψ_1 and ψ_2 and depict the graph associated with these potentials.
- [2 pts.] Reconstruct p .
- [3 pts.] Does this graph represent the independency model of p ? Justify your answer.
- [3 pts.] Are B and C conditionally independent given D in p ?
- [3 pts.] Does this graph represent the independency model of p faithfully?

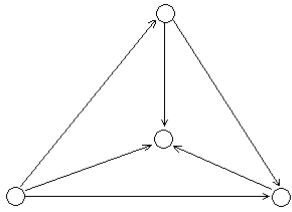


Fig.1

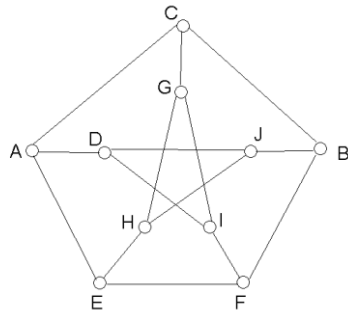


Fig.2 (Petersen Graph)

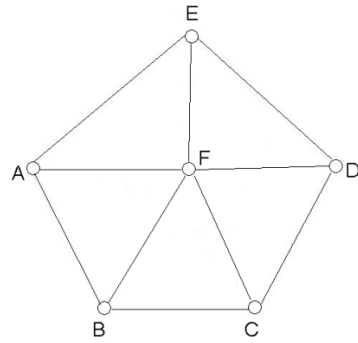


Fig.3.

A	B	P(A,B)
0	0	0.2
0	1	0.2
1	0	0.3
1	1	0.3

Table 1.

B	C	D	P(A,B,C)
0	0	0	0.225
0	0	1	0.0666...
0	1	0	0.075
0	1	1	0.0333...
1	0	0	0.075
1	0	1	0.3333...
1	1	0	0.025
1	1	1	0.1666...

Table 2.