Tutorial 5

Solutions should be given till 26th November 2007, 16:00

In several practical cases **joint probability distributions (JDP)** are useful for inference (either direct, or indirect i.e. by building a model based on JDP). Some such inference tasks may be the prediction of the risk of an illness, or the prediction of the credit score of a customer of a bank. If we have lot of variables and we just want to store the JDP in a "naive" way, we need to much space. For the efficiency we want to **factorise the JDP**. According to [lemma 6 at slide 25 in bayes-04-independence-markov.pdf] we can factorise a JDP if we find a **chain of cliques** [slide 20] in the graph representation [slides 6-19] of the independence model. If we have a **perfect ordering** [slide 22] we can easily find a chain of cliques. But how can we find a perfect ordering? We have an algorithm [slide 23-24] for this task, if we have a **triangulated graph** [slide 21].
Although we can factorise a JDP according to the cliques of the graphical representation (Markov network) of the independence model, the potentials used for this factorisation are not intuitive (when the graph is not triangulated): unfortunately the potentials do not have natural (human "understandable") interpretation. Thus we began studying another graphical representation of the independence model, called **Bayesian Networks** [bayes-05-independence-bayesian.pdf].

Exercise 1 Topological order (10 points)

a) [5 pts.] Modify the graph in Figure 2 with addition of some edges so that it will be chordal (triangulated)!
b) [5 pts.] Does the modified graph have a chain of cliques? And the original graph?

Exercise 2 Perfect ordering (10 points + 5 bonus points)

a) [5 pts.] Let *G* denote a complete graph with *n* vertexes. How many different perfect orderings does *G* have?
b) [5 pts.] Does the graph in
    a. Figure 1
    b. Figure 2

   have a perfect ordering? If yes, show a perfect ordering. If not, prove why it does not have any perfect ordering.
c) [optional, 5 bonus pts.] The algorithm in slide 23 of bayes-04-independence-markov.pdf finds a perfect ordering if the input graph *G* has any. However, if the input graph *G* does not have any perfect ordering, this algorithm will just output some non-perfect ordering, without any error message or throwing an exception. Modify the algorithm so that it stops (throws an exception/writes an error message/terminates) if G does not have any perfect ordering!

Exercise 3 Bayesian Networks (10 Points)

Suppose we are given the following independence model:

I={	i(A, B | C),	i(B, A | C),	i(A, B | D),	i(B, A | D),
	i(A,{B C} | D),	i({B,C}, A | D),	i(A, C | D),	i(C, A | D),
	i(A, B | {C, D}),	i(B, A | {C, D}),	i(A, C | {B, D}),	i(C, A | {B, D})
	i(A, {B,C} | D),	i({B,C}, A | D)	}	

a) [6 pts.] What is the minimal directed representation (Bayesian network) of this independence model, if the ordering of the vertexes is
   a. A, B, C, D
   b. D, C, B, A ?

   Is it unique?

b) [4 pts.] Factorize the JDP *p*(A,B,C,D) according to the Bayesian networks constructed in the previous task.

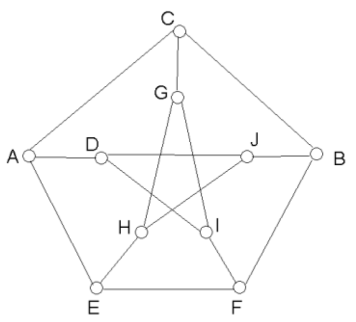

Fig.1. (Petersen Graph)

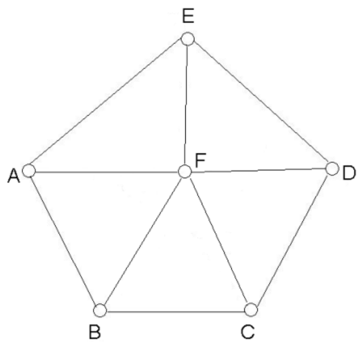

Fig.2.