

## Übung 2

### Grundlagen Graphen-Theorie

Lösungen sind via Moodle einzureichen

#### Aufgabe 1 u-separation (10 Points)

- [5 pts.] The graph in Figure 1 is given. What is the smallest set  $Z$  (i.e. the set with minimal amount of vertexes) so that  $\{A\}$  and  $\{B\}$  are u-separated?
- [5 pts.] The graph in Figure 2 is given. Are the sets of nodes  $\{A,B\}$  and  $\{F,G,H\}$  u-separated by  $Z$  ?
  - $Z=\{C,D\}$
  - $Z=\{C,D,E\}$

#### Aufgabe 2 Blocking (10 Points)

- [2 pts.] Consider the graph in Figure 3. Which are the ancestors and descendents of  $\{J,I,D\}$ ,  $\{L,F\}$  and  $\{L,M\}$  ?
- [5 pts.] At which positions is the chain  $A,B,C,D,E,F,G,H$  blocked by  $\{I,D,J\}$ ,  $\{L,F\}$  and  $\{L,M\}$
- [3 pts.] Is it possible to choose directions for all edges of the graph in Figure 1, so that the empty set does not block any node of any chain?

#### Aufgabe 3 d-separation (10 Points)

- [3 pts.] Consider the graph in Figure 4. Construct the moral graph of this graph!
- [3 pts.] Are the sets  $\{A,B,C,D,E\}$  and  $\{I,J,K,L\}$  d-separated by  $\{F,G,H\}$  in the original graph? (Please answer this question based on the moral graph.)
- [4 pts.] Are the sets  $\{A,B,C,D,E\}$  and  $\{I,J,K,L\}$  d-separated by  $\{F,G,H\}$  in the original graph? (Please answer this question based on the "first" definition of d-separation.)

#### Aufgabe 4 Pfade (10 Points)

- [2 pts.] Consider the graph in Figure 1. We say, two paths are disjoint, if the corresponding sequences do not have any common edge. For example  $(E,H,G,C,A,D)$  and  $(E,F,I,D)$  are two disjoint paths. What is the amount of disjoint paths between  $A$  and  $B$ ? We do not allow cycles in the path (the same vertex should only be used once.)
- [8 pts.] Two paths are said to be different if their sequences are not the same. For example the paths  $\{A, C, B\}$  and  $\{A, C, G, I, F, B\}$  are different. (They have common elements, but the whole sequences are not the same.) What is the amount of different paths between  $A$  and  $B$ ? We do not allow cycles in the path (the same vertex should only be used once.) (Please explain your answer in details.)

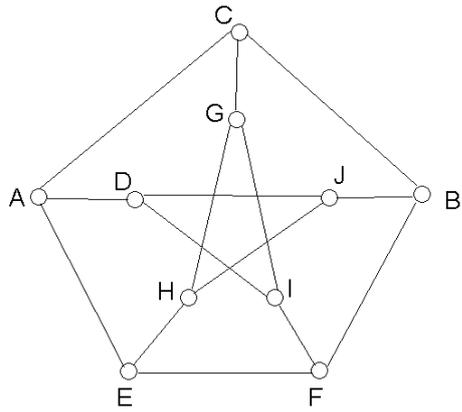


Fig.1 Petersen Graph

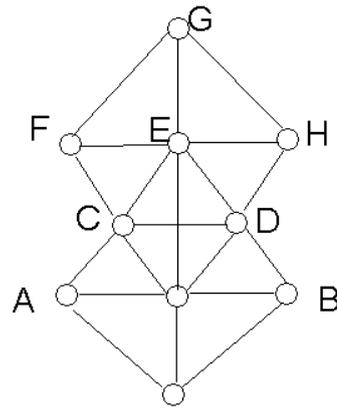


Fig. 2

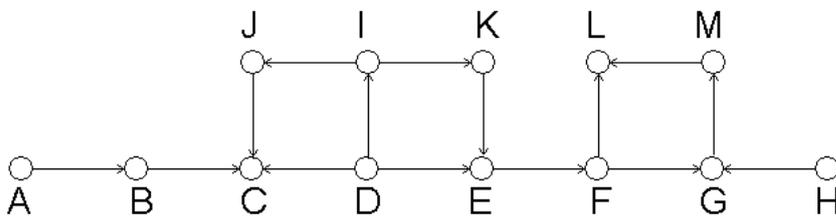


Fig. 3

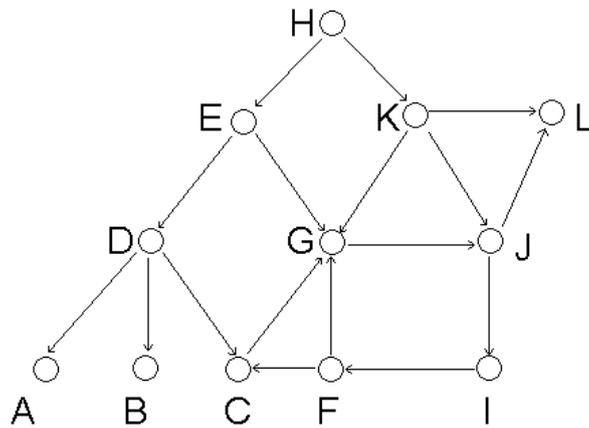


Fig. 4