Übung 3

Markov Networks (1.)

Lösungen bitte via Moodle / learnweb einreichen

Aufgabe 1   Topological order   (10 points)

a) [3 pts.] Does the graph in Figure 1 have a topological order? If yes, is this unique? (Does it have only one topological order or more?)
b) [2 pts.] Show an example for a graph, which has several topological orders!
c) [5 pts.] Modify the graph in Figure 3 with addition of some edges so that it will be chordal (triangulated)!

Exercise 2   Graphical representation of independence   (10 Points)

d) [3 pts.] Is the graph in Fig. 3 a representation of the following independence model? I={I(A,C|{E,F,B}), I(C,A|{E,F,B}), I(A,C|{D,F,B}), I(C,A|{D,F,B}), I(A,D|{E,F,B}), I(D,A|{E,F,B}), I(A,D|{E,F,C}), I(D,A|{E,F,C})}
e) [1 pts.] Is the graph in Fig. 3 a <u>faithful</u> representation of the indep. relation above?
f) [2 pts.] Construct (another) graph, which represents the independence relation above! (This graph should not necessary be a faithful representation.)
g) [4 pts.] For which independence relation is the graph in Fig. 3 a faithful representation?

Exercise 3   Properties of independency models, graphical representation   (10 Points)

Suppose we are given the following independence model:
I(A,B|{C,D}), I(B,A|{C,D}), I(A,C|{D,E})

a) [3 pts.] Which of the properties (symmetric, decomposable and intersectable) hold for this model?
b) [2 pts.] Modify the model so that all of the properties above hold for the model!
c) [5 pts.] Construct the minimal undirected graph representation of the model. Is it trivial? Is it unique? Is it faithful?

Exercise 4   Potentials, Markov Networks   (10 Points)

Given that the potentials ψ_1, ψ_2 represented in Table 1 and Table 2 factorize the JPD p, solve the following tasks:
a) [4 pts.] Multiply ψ_1 and ψ_2 and depict the graph associated with these potentials.
b) [2 pts.] Reconstruct p.
c) [2 pts.] Does this graph represent the independency model of p? Justify your answer.
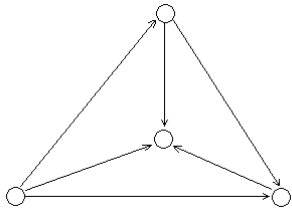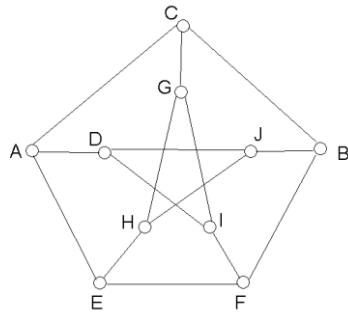d) [2 pts.] Are B and C conditionally independent given D in p? Why?
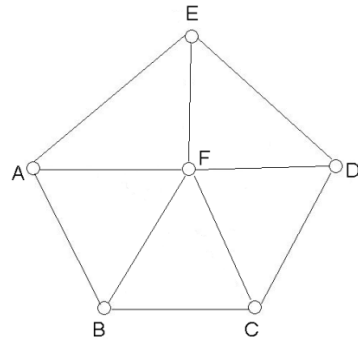
Fig.1   Fig.2 (Petersen Graph)   Fig.3.

A	B	P(A,B)
0	0	0.2
0	1	0.2
1	0	0.3
1	1	0.3

Table 1.

B	C	D	P(A,B,C)
0	0	0	0.225
0	0	1	0.0666...
0	1	0	0.075
0	1	1	0.0333...
1	0	0	0.075
1	0	1	0.3333...
1	1	0	0.025
1	1	1	0.1666...

Table 2.