

# Bayesian Networks - Exercise Sheet 1

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Solutions need to be handed in until **Tuesday, November 5th, 2013 at 08:00**

## Exercise 1: Events and Conditional Probabilities (12P)

- a) Consider two distinguishable dices that are being rolled simultaneously. Let  $\Omega := \{1, \dots, 6\}^2$  be the event space of all  $(i, j)$  tuples. Let  $X : \Omega \rightarrow \mathbb{N}$  be a random variable defined by:

$$X : (i, j) \mapsto i + j$$

For the upcoming examples, write down their set of elementary events (all  $e \in \Omega$ ) and compute the probability. Note that not all events need to involve  $X$ .

- $A$  : Rolling the same number twice
- $B$  :  $X < 4$
- $C$  :  $X \leq 4$
- $D$  :  $X$  is an even number
- $E$  : The first dice shows the number 2

Now compute the following conditional probabilities:

- $P(D|A)$  and  $P(A|D)$
- $P(C|A)$  and  $P(A|C)$
- $P(E|C)$  and  $P(C|E)$
- Are  $A$  and  $C$  conditionally independent given  $B$ ? Explain your answer in detail.

- b) A Skat Game consists of 16 red cards (hearts, diamonds) and 16 black cards (spades, clubs). Of every color there are 8 cards which are in descending order ace, king, queen, jack, 10, 9, 8, 7. Let the event  $A$  denote *drawing an ace out of a full stack of cards*. Your friend Tom draws cards and gives hints. Do the hints help in deducing if he drew an ace? Explain your answers in detail.

- Tom draws a card and says it is of clubs.

- Tom draws a card and says it is not a queen.
- Tom draws a card and says it is red.
- Tom draws a card and says the card is lower than a jack.

**Exercise 2: Marginalizing a Joint Probability Distribution (8P)**

The cold is an annoying disease and has very many symptoms. Three of these symptoms are *nausea*, a *sore throat* and a *headache*. In the table below, there are the results of 280 patients.

- Copy the table from below and write down the relative probabilities, i.e. make a table of the joint probability distribution  $P(C, N, S, H)$ .
- Write down the following marginal distributions  $P(N)$ ,  $P(S)$  and  $P(H)$ .
- How many patients were suffering from a cold but did not suffer from headaches?
- Which variable is more expressive for diagnosing a cold, is it nausea or headache?

		Sore Throat				Nausea			
		Yes		No		Yes		No	
		Yes	No	Yes	No	Yes	No	Yes	No
Cold	Yes	67	12	3	37	25	3	1	1
	No	2	5	6	17	12	20	14	55