

Bayesian Networks - Exercise Sheet 3

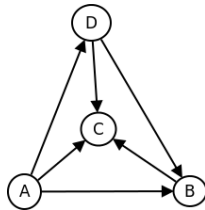
Lydia Voß
voss@ismll.de

November 9, 2016

Solutions need to be handed in until **Monday, November 14th 12:00pm**

Exercise 1: Orderings (10 P)

- a) Assume we are given the following graph. Does a topological ordering exist and if it exists is it unique?



- b) Does a directed graph always have **exact one** topological ordering? If not, draw a graph with 4 different topological orderings and write these down.
- c) Let $G_1 = (V_1, E_1)$ be the directed graph of figure 1. Is the following ordering a topological ordering? Explain your answer.

$$\sigma_1 = (A, B, C, D)$$

$$\sigma_2 = (A, B, D, C)$$

$$\sigma_3 = (A, C, B, D)$$

$$\sigma_4 = (B, A, D, C)$$

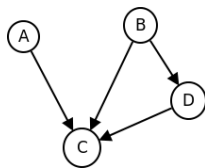


Figure 1: DAG G_1

- d1) Let $G_2 = (V_2, E_2)$ be the directed graph from figure 2. Does a topological ordering exist? Write down an **equivalent** condition for the existence of a topological ordering.

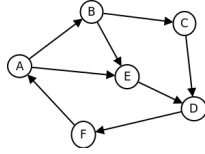


Figure 2: Graph G_2

- d2) Which edges need to be deleted (just **one**) so that G_2 has a topological ordering?

Exercise 2: Graphical representation of ternary relations (10 P)

Given is the graph G_I in figure 3. Let I be the ternary relation with:

$$I = \left\{ (A, C | \{E, F, B\}), (C, A | \{E, F, B\}), (A, C | \{D, F, B\}), \right. \\ \left. (C, A | \{D, F, B\}), (A, D | \{E, F, B\}), (D, A | \{E, F, B\}) \right\}$$

- Is I a symmetric relation?
- Is G_I a representation of I ? If yes, is it a faithful representation?
- Draw a graph which represents I . It must not necessarily be a faithful one, but **shouldn't** be a trivial one.
- Write down a ternary representation of I for which G_I is a faithful representation (you can ignore symmetry).

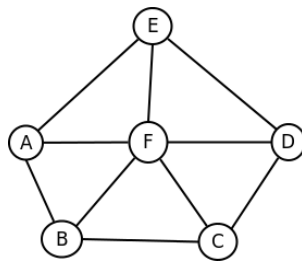


Figure 3: The undirected graph G_I

e) Let \hat{I} be another ternary relation with

$$\hat{I} = \left\{ (A, B | \{C, D\}), (B, A | \{C, D\}), (A, C | \{D, B\}) \right\}$$

Which properties *symmetry*, *decomposability* and *separation* does \hat{I} have? Extend \hat{I} so that it has all properties.