

## Bayesian Networks - Exercise Sheet 4

Solutions need to be handed in until **Tuesday, November 22nd 2pm**

### Exercise 1: Markov Networks, Potentials (10 Points)

Given are two potentials  $\psi_1$  and  $\psi_2$  that factorize a joint probability distribution  $P(A, B, C, D)$ .

$A$	$B$	$P(A, B)$
0	0	0.2
0	1	0.2
1	0	0.3
1	1	0.3

Table 1: Potential  $\psi_1$

$D$	$B$	$C$	$P(B, C, D)$
0	0	0	0.225
0	0	1	0.05
0	1	0	0.075
0	1	1	0.15
1	0	0	0.075
1	0	1	0.15
1	1	0	0.025
1	1	1	0.25

Table 2: Potential  $\psi_2$

- a) Multiply both potentials and reconstruct  $P$ . Normalize  $P$ . What is the normalization constant?
- b) Sketch the graph  $G$  that results from the factorization. List all independencies that are visible in the graph.
- c) Proof that  $I_P(A, C | B)$  holds.
- d) Does the independence relation  $I_P(B, D | C)$  hold? Explain.
- e) Is  $G$  a representation of  $P$ ? If yes is  $G$  a faithful representation of  $P$ ? Explain.

### Exercise 2: Cliques in Graphs, Triangulation (10 Points)

Have a look at the undirected graph in Figure 1.

- a) List all cliques of  $G$ . Does  $G$  contain a chain of cliques? Explain.

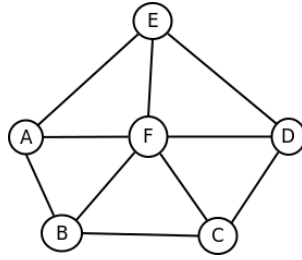


Figure 1: The undirected graph  $G$ .

- b) Modify  $G$  (by adding edges) to a triangulated graph  $G^{\text{mod}}$ . List all cliques of  $G^{\text{mod}}$ . Does  $G^{\text{mod}}$  contain a chain of cliques? Explain.
- c) Does  $G$  or  $G^{\text{mod}}$  contain a perfect ordering? Name the perfect ordering, if it exists. Explain.