Bayesian Networks - Exercise Sheet 1

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Solutions need to be handed in until **Monday, April 16th, 2018 at 12:00**

Exercise 1: Events and Conditional Probabilities (12P)

a) Consider two distinguishable dices that are being rolled simultaneously. Let $\Omega := \{1, ..., 6\}^2$ be the event space of all (i, j) tuples. Let $X : \Omega \to \mathbb{N}$ be a random variable defined by:

$$X : (i, j) \mapsto i + j$$

For the upcoming examples, write down their set of elementary events (all $e \in \Omega$) and compute the probability. Note that not all events need to involve X.

- A : Rolling the same number twice
- $B : X < 4$
- $C : X \leq 4$
- $D : X$ is an even number
- E : The first dice shows the number 2

Now compute the following conditional probabilities:

- $P(D|A)$ and $P(A|D)$
- $P(C|A)$ and $P(A|C)$
- $P(E|C)$ and $P(C|E)$
- Are A and C conditionally independent given B? Explain your answer in detail.

b) A Skat Game consists of 16 red cards (hearts, diamonds) and 16 black cards (spades, clubs). Of every color there are 8 cards which are in descending order ace, king, queen, jack, 10, 9, 8, 7. Let the event A denote *drawing an ace out of a full stack of cards*. Your friend Tom draws cards and gives hints. Do the hints help in deducing if he drew an ace? Explain your answers in detail.

- Tom draws a card and says it is of clubs.

- Tom draws a card and says it is not a queen.
- Tom draws a card and says it is red.
- Tom draws a card and says the card is lower than a jack.

Exercise 2: Marginalizing a Joint Probability Distribution (8P)

The cold is an annoying disease and has very many symptoms. Three of these symptoms are *nausea*, a *sore throat* and a *headache*. In the table below, there are the results of 280 patients.

a) Copy the table from below and write down the relative probabilities, i.e. make a table of the joint probability distribution $P(C, N, S, H)$.

b) Write down the following marginal distributions $P(N), P(S)$ and $P(H)$.

c) How many patients were suffering from a cold but did not suffer from headaches?

d) Which variable is more expressive for diagnosing a cold, is it nausea or headache?

Sore Throat	Yes				No			
Nausea	Yes		No		Yes		No	
Headache	Yes	No	Yes	No	Yes	No	Yes	No
Cold								
Yes	67	12	3	37	25	3	1	1
No	2	5	6	17	12	20	14	55