## Networks in Their Surrounding Contexts

## Objectives

- Examine additional processes (to triadic closure) that affect the formation of links in the network
- Surrounding contexts: factors that exist outside the nodes and edges of a network
- Represent the contexts together with the network in a common framework


## Homophily

- Homophily principle: we tend have similar characteristics with our friends



## "birds of a feather flock together"



- People of similar character, background, or taste tend to congregate or associate with one another (like likes like)
- expression appears in the 16th century, a literal translation of Plato's Republic


## Homophily

- Links in a social network tend to connect people who are similar to one another
- basic notions governing the structure of social networks
- Its role in modern sociology by influential work in the 1950s (Lazarsfeld and Merton)


## Homophily vs. Triadic Closure for Link Formation

- With triadic closure:
- a new link is added for reasons that are intrinsic to the network (need not look beyond the network)
- Ex: a friendship that forms because two people are introduced through a common friend
- With homophily:
- a new link is added for reasons that are beyond the network (at the contextual factors)
- Ex: a friendship that forms because two people attend the same school or work for the same company


## Example



Social network from a town's middle school and high school (students of different races drawn as differently colored circles)
2 divisions:

- one based on race and
- the other based on friendships in the middle and high schools


## Homophily vs. Triadic Closure for Link Formation

- Strong interactions between intrinsic and contextual effects
- Both operating concurrently
- Triadic closure (intrinsic mechanism):
$-B$ and $C$ have a common friend $A$
$-B$ and $C$ have increased opportunities to meet
- Homophily (contextual mechanism):
- $B$ and $C$ are each likely to be similar to $A$ in a number of dimensions
- also possibly similar to each other as well
- Most links arise from a combination of several mechanisms
- difficult to attribute any individual link to a single mechanism


## Measuring Homophily

- Given a characteristic (like race, or age), how to test if a network exhibits homophily according to it?
- Ex friendship network:
- Exhibits homophily by gender?
- boys tend to be friends with boys, and girls tend to be friends with girls
- cross-gender edges exist

friendship network of a (hypothetical) classroom: shaded nodes are girls and the six unshaded nodes are boys


## Measuring Homophily

- Q: what would it mean for a network not to exhibit homophily by gender?
- A: number of crossgender edges not very different from randomly assigning each node a gender
- according to the gender balance in the original network



## Measuring Homophily

- $p$ the probability (fraction) of males
- $q=1-p$ the probability (fraction) of females
- For a given edge:
- Homophily:
- $\operatorname{Prob}\left(b o t h\right.$ ends male) $=p^{*} p$
- $\operatorname{Prob}\left(b o t h\right.$ ends female) $=q^{*} q$
- Cross gender:
- $\operatorname{Prob}(e n d s$ male and female) $=$ 2*p*q
- Homophily Test: If the fraction of cross-gender edges is significantly less than $2 p q$, then there is evidence for homophily



## Measuring Homophily

- Ex:
- $p=6 / 9=2 / 3$
- $q=1 / 3$
- $2 p q=4 / 9=8 / 18$
- 5/18 cross-gender edges
- Test: $5 / 18<8 / 18$ => some evidence of homophily
- Need definition of "significantly less than"
- standard statistical significance
- What if cross-gender edges more than $2 p q$ ?

- inverse homophily (Ex: network of romantic relationships)
- How to extend to characteristics with more than 2 states?


## Mechanisms Underlying Homophily

- Homophily has 2 mechanisms for link creation
- Selection: select friends with similar characteristics
- individual characteristics drive the formation of links
- involves immutable characteristics (determined at birth)
- Social influence: modify behavior close to behaviors of friends
- the reverse of selection
- involves mutable characteristics (behaviors, activities, interests, beliefs, and opinions)


## The Interplay of Selection and Social Influence

- Q: When homophily is observed, is it a result of selection or social influence?
- Have people adapted their behaviors to become more like their friends, or have they selected friends who were already like them?
- A: Track the network and monitor the results of the two mechanisms (more details later)


## The Interplay of Selection and Social Influence

- Most of the times, both mechanisms apply and interact with each other
- Studies show that teenage friends are similar to each other in their behaviors, and both selection and social influence apply:
- teenagers seek social circles of people like them and peer pressure causes conform to behavioral patterns within these circles
- Q: how the two mechanisms interact and whether one is more strongly at work than the other? (more details later)


## Affiliation

- Story so far:
- Homophily groups together similar nodes
- Selection and social influence determine the formation of links in a network
- Similarity of nodes based on characteristics
- How to model these characteristics?
- They represent surrounding contexts of networks
- They exist "outside" the network
- How to put these contexts into the network itself?


## Affiliation

- Represent the set of activities a person takes part in (a general view of "activity")
- Ex: part of a particular company, organization, frequenting a particular place, hobby
- Refer to activities as foci: "focal points" of social interaction


## Affiliation Networks

- Affiliation network:
- bipartite graph
- nodes divided into 2 sets
- no edges joining a pair of nodes that belong to the same set
- people affiliated with foci
- Ex:
- Anna participates in both of the social foci on the right
- Daniel participates in only one



## Co-Evolution of Social and Affiliation Networks

- Social networks change over time
- new friendship links are formed
- Affiliation networks change over time
- people become associated with new foci
- Co-evolution reflects interplay between selection and social influence
-2 people participate in a shared focus can become friends
- if 2 people are friends, they can share their foci
- How to represent co-evolution with a single network?


## Social-affiliation networks

- Social-affiliation network contains:
- a social network on the people and
- an affiliation network on the people and foci



## Social-affiliation networks

- In social-affiliation networks link formation as a closure process
- Several options for "closing" B-C
- triadic closure: $\mathrm{A}, \mathrm{B}$, and C represent a person (already examined)
- focal closure: B and C people, A focus
- selection: B links to similar C (common focus)
- membership closure: A and B people, C focus
- social influence: $B$ links to C influenced by A

person
(a) Iriadic closure

focus
(b) Fooul closure

(c) Membership closure


## Example

- Bob introduces Anna to Claire
- Karate "introduces" Anna to Daniel
- Anna introduces

Bob to Karate


Edges with bold are the newly formed

## Tracking Link Formation in On-Line Data

- Story so far: a set of mechanisms that lead to the formation of links
- triadic closure
- focal closure
- membership closure
- Tracking these mechanisms in large populations
- their accumulation observable in the aggregate


## Tracking triadic closure

- Likelihood of link as a function of common friends?

1. Two snapshots of the network
2. For each $k$, find all pairs of nodes with $k$ common friends in the first snapshot, but not directly connected
3. $T(k)$ : fraction of these pairs connected in the second snapshot

- empirical estimate of probability that a link will form between two people with $k$ common friends

4. Plot $T(k)$ as a function of $k$

- $\quad T(0)$ is the rate of link formation when it does not close a triangle


## Tracking triadic closure

- Kossinets and Watts computed T(k)
- full history of e-mail communication ("who-talks-to-whom")
- a one-year period
$-22,000$ students at a large U.S. university
- observations in each snapshot were one day apart (average over multiple snapshots)


## Tracking triadic closure

- Interpret the result compared to a baseline
- Assume that each common friend that 2 people have, gives them an independent probability $\mathbf{p}$ of forming a link
-2 people have $k$ friends in common $=>$ the probability they fail to form a link is (1-p)^k
-2 people have $k$ friends in common => probability that they form a link is $\mathbf{1 - ( 1 - p )}{ }^{\wedge} \mathbf{k}$


## Tracking triadic closure



## Tracking focal closure

- Likelihood of link formation as a function of the number of common foci?
- Kossinets and Watts supplemented their university e-mail dataset with information about the class schedules
- each class became a focus
- students shared a focus if they had taken a class together


## Tracking focal closure



## Tracking membership closure

- Blogging site LiveJournal
- social network (friendship links)
- foci correspond to membership in user-defined communities

probability of joining a LiveJournal community as a function of the number of friends who are already members


## Tracking membership closure

- Wikipedia editors
- link editors when they communicated (user talk page)
- each Wikipedia article defines a focus (editor associated with the articles he/she edited)

probability of editing a Wikipedia articles as a function of the number of friends who have already done so


# Quantifying the Interplay Between Selection and Social Influence 

- How selection and social influence work together to produce homophily?
- How do similarities in behavior between two Wikipedia editors relate to their pattern of social interaction over time?
- Similarity between 2 Wikipedia editors A, B:


## number of articles edited by both $A$ and $B$

number of articles edited by at least one of $A$ or $B$

- Is homophily (similarity) due to editors connected (talk) with those edited the same articles (selection), or because editors are led to edit articles by those they talk to (social influence)?


# Quantifying the Interplay Between Selection and Social Influence 


"tick" in time whenever either A or B performs an action (editing or talking).
Time 0 is the point at which they first talked

## A SPATIAL MODEL OF SEGREGATION

## Spatial patterns of segregation

- One of the most strong effects of homophily is in the formation of ethnically and racially homogeneous neighborhoods in cities
- a process with a dynamic aspect
- what mechanisms?

(a) Chicago, 1940

(b) Chicago, 1960

In blocks colored yellow and orange the percentage of African-Americans is below 25, while in blocks colored brown and black the percentage is above 75

## The Schelling Model

- How global patterns of spatial segregation can arise from the effect of homophily operating at a local level (Thomas Schelling)
- an intentionally simplified mechanism
- works even when no one individual explicitly wants a segregated outcome


## The Schelling Model

- Model assumptions:
- Population of individuals called agents
- Each agent of type X or type O
- The two types represent some characteristic as basis for homophily (race, ethnicity, country of origin, or native language)
- Agents reside in cells of a grid (simple model of a 2-D city map)
- Some cells contain agents while others are unpopulated
- Cell's neighbors: cells that touch it (including diagonal contact)

| $x$ | $x$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | 0 |  | 0 |  |  |
| $x$ | $x$ | 0 | 0 | 0 |  |
| $x$ | 0 |  |  | $x$ | $x$ |
|  | 0 | 0 | $x$ | $x$ | $x$ |
|  |  | 0 | 0 | 0 |  |

## The Schelling Model

| $x$ | $x$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | 0 |  | 0 |  |  |
| $x$ | $x$ | 0 | 0 | 0 |  |
| $x$ | 0 |  |  | $x$ | $x$ |
|  | 0 | 0 | $x$ | $x$ | $x$ |
|  |  | 0 | 0 | 0 |  |

(a) Agents occupying cells on a grid.

(b) Neighbor relations as a graph.

Cells are the nodes and edges connect neighboring cells. We will continue with the geometric grid rather than the graph.

## The Schelling Model

- Local mechanism:
- each agent wants to have at least some t other agents of its own type as neighbors ( $t$ the same for all)
- unsatisfied agents have fewer than $t$ neighbors of the same type as itself and move to a new cell
- Ex (figure):
- agents with ID
$-\mathrm{t}=3$

| $\mathrm{X} 1^{\text {² }}$ | X2 ${ }^{\text { }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X3 | O1* |  | O2 |  |  |
| X4 | X5 | O3 | O4 | O5 ${ }^{\text {x }}$ |  |
| X6 ${ }^{\text { }}$ | O6 |  |  | X7 | X8 |
|  | O7 | O8 | X9* | X10 | X11 |
|  |  | O9 | 010 | O11 ${ }^{\text { }}$ |  |

(a) An initial configuration.

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X 3 | X 6 | O 1 | O 2 |  |  |
| $\mathrm{X4}$ | X 5 | O 3 | O 4 |  |  |
|  | O 6 | X 2 | X 1 | X 7 | X 8 |
| O 11 | O 7 | O | $\mathrm{X9}$ | X 10 | X 11 |
|  | O | O | O |  |  |

## The Dynamics of Movement

- Unsatisfied agents move in rounds
- consider unsatisfied agents in some order
- random or row-sweep
- unsatisfied agents move to an unoccupied cell where will be satisfied
- random or to nearest cell that satisfies them
- may cause other agents to be unsatisfied
- deadlocks may appear (no cell that satisfies)
- stay or move randomly
- All variations have similar results
- Ex (figure):
- t=3, one round, row-sweep, move to nearest cell, stay when deadlocks

| X1 ${ }^{\text {* }}$ | X2 ${ }^{\text {x }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X3 | O1* |  | O2 |  |  |
| X4 | X5 | O3 | O4 | O5 ${ }^{\text { }}$ |  |
| X6 ${ }^{\text {² }}$ | 06 |  |  | X7 | X8 |
|  | O7 | O8 | X9* | X10 | X11 |
|  |  | O9 | 010 | O11 ${ }^{\text {² }}$ |  |

(a) An initial configuration.

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :--- |
| X 3 | X 6 | O 1 | O 2 |  |  |
| X 4 | X 5 | O 3 | $\mathrm{O4}$ |  |  |
|  | O6 | X 2 | X 1 | X 7 | X 8 |
| O11 | O7 | O8 | $\mathrm{X9}$ | X 10 | X 11 |
|  | O5 | O9 | $\mathrm{O} 10^{x}$ |  |  |

(b) After one round of movement.

## Larger examples


(a) A simulation with threshold 3.

(b) Another simulation with threshold 3.

Two runs ( 50 rounds) of the Schelling model with unsatisfied agents moving to a random location. Threshold $\mathrm{t}=3,150-\mathrm{by}$-150 grid with 10, 000 agents. Each cell of first type is red, of second type blue, or black if unoccupied.

## Interpretations of the Model

- Spatial segregation is taking place even though no individual agent is seeking it
- agents just want to be near $t$ others like them
- when $t=3$, agents are satisfied being minority among its neighbors ( 5 neighbors of the opposite type)
- Ex (figure):
- a checkerboard $4 \times 4$ pattern can make all agent satisfied (even for

| $x$ | $x$ | 0 | 0 | $x$ | $x$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $x$ | $x$ | 0 | 0 | $x$ | $x$ |
| 0 | 0 | $x$ | $x$ | 0 | 0 |
| 0 | 0 | $x$ | $x$ | 0 | 0 |
| $x$ | $x$ | 0 | 0 | $x$ | $x$ |
| $x$ | $x$ | 0 | 0 | $x$ | $x$ | large grids)

- we don't see this result in simulations


## Interpretations of the Model

- More typically, agents form larger clusters
- agents become unsatisfied and attach to larger clusters (where higher probability to be satisfied)
- The overall effect:
- local preferences of individual agents have produced a global pattern that none of them necessarily intended


## Interpretations of the Model


(a) After 20 steps

(c) After 350 steps

(b) After 150 steps

(d) After 800 steps
$\mathrm{t}=4,150-$ by-150 grid, 10, 000 agents, varying number of rounds (steps), not shown until the end

## Schelling model and Homophily

- The Schelling model is an example that, as homophily draws people together along immutable characteristics (race or ethnicity), it creates a natural tendency for mutable characteristics (decision about where to live) to change in accordance with the network structure

