# Outline







# Outline

# Topics of the Lecture

1. Simultaneous Localization and Mapping from Video (Visual SLAM)

2. Image Classification and Description





# Simultaneous Localization and Mapping


[source https://www.youtube.com/watch?v=bDOnn0-4Nq8]

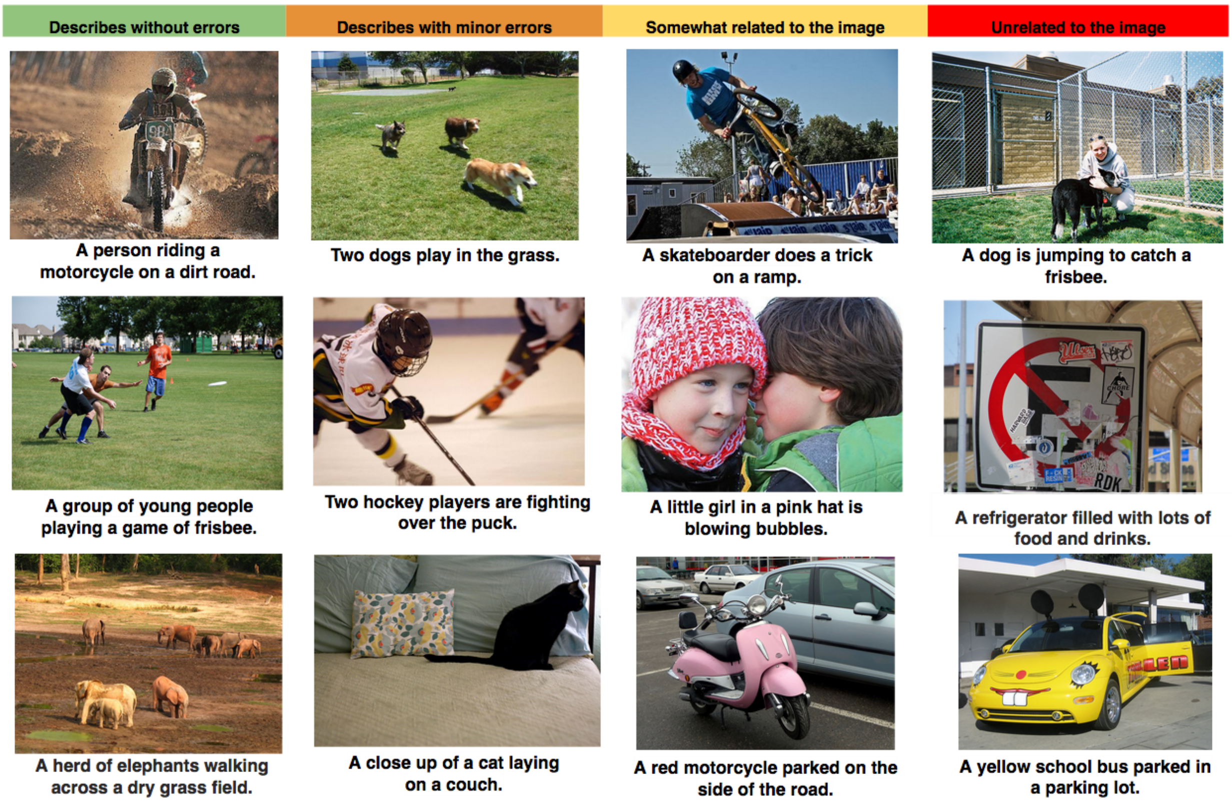
# Simultaneous Localization and Mapping from Video

- SLAM usually employs laser range scanners (lidars).
- **Visual SLAM**: use video sensors (cameras).
- main parts required:
  1. Projective Geometry
  2. Point Correspondences
  3. Estimating Camera Positions (Localization)
  4. Triangulation (Mapping)





# Image Classification and Description




[source: http://googleresearch.blogspot.de/2014/11/a-picture-is-worth-thousand-coherent.html]

# Outline







# Motivation

In Euclidean (planar) geometry, there are many exceptions, e.g.,

- most two lines intersect in exactly one point.
- but some two lines do not intersect.
  - parallel lines

Idea:

- add **ideal points**, one for each set of parallel lines / direction
- define these points as intersection of any two parallel lines
- now any two lines intersect in exactly one point
  - either in a finite or in an ideal point

# Homogeneous Coordinates: Points

Inhomogeneous coordinates:

$$x \in \mathbb{R}^2$$

Homogeneous coordinates:

$$x \in \mathbb{P}^2 := \mathbb{R}^3 / \equiv$$

$$x \equiv y :\Longleftrightarrow \exists s \in \mathbb{R} \setminus \{0\} : sx = y, \quad x, y \in \mathbb{R}^3$$

Example:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \equiv \begin{pmatrix} 4 \\ 8 \\ 12 \end{pmatrix} \text{ represent the same point in } \mathbb{P}^2$$

$$\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \text{ represent a different point in } \mathbb{P}^2$$





# Homogeneous Coordinates: Lines

Inhomogeneous coordinates:

$$a \in \mathbb{R}^3 : \ell_a := \{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid a_1 x_1 + a_2 x_2 + a_3 = 0 \}$$

- $a_1 \neq 0$ or $a_2 \neq 0$ (or both $a_1, a_2 \neq 0$).
- $sa = (sa_1, sa_2, sa_3)^T$ encodes the same line as $a$ (any $s \in \mathbb{R}, s \neq 0$).

Homogeneous coordinates:

$$a \in \mathbb{P}^2 : \ell_a := \{ x \in \mathbb{P}^2 \mid a^T x = a_1 x_1 + a_2 x_2 + a_3 x_3 = 0 \}$$

- contains all finite points of $a' \in \kappa^{-1}(a)$: $\ell_{\kappa(a')} \supsetneq \iota(\ell_{a'})$
- and the ideal point $(a_2, -a_1, 0)^T$.
  - intersection of parallel lines (same $a_1, a_2$, different $a_3$)

Note: $\kappa : \mathbb{R}^3 \rightarrow \mathbb{P}^2, a \mapsto [a] := \{ a' \in \mathbb{R}^3 \mid a' \equiv a \}$.

# A point on a line

A point $x$ lies on line $a$ iff $x^T a = 0$.





# Intersection of two lines

Lines $a$ and $b$ intersect in $a \times b := \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ -a_1 b_3 + a_3 b_1 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

Proof:

$$a^T(a \times b) = a_1 a_2 b_3 - a_1 a_3 b_2 - a_2 a_1 b_3 + a_2 a_3 b_1 + a_3 a_1 b_2 - a_3 a_2 b_1 = 0$$
$$b^T(a \times b) = \ldots = 0$$

Example:

$$\begin{aligned} x = 1 : & a = (-1, 0, 1)^T \\ y = 1 : & b = (0, -1, 1)^T \\ a \times b = & (1, 1, 1)^T \end{aligned}$$

Esp. for parallel lines: $b_1 = a_1, b_2 = a_2, b_3 \neq a_3$:

$$a \times b \equiv \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix}$$

# Line joining points

The line through $x$ and $y$ is $x \times y$.

Proof: exactly the same as previous slide.

Example:

$$\begin{aligned} x =& (-1, 0, 1)^T \\ y =& (0, -1, 1)^T \\ x \times y =& (1, 1, 1)^T \end{aligned}$$





# Line at infinity

All ideal points form a line:

$$l_\infty := (0, 0, 1)^T \quad \textbf{line at infinity}$$

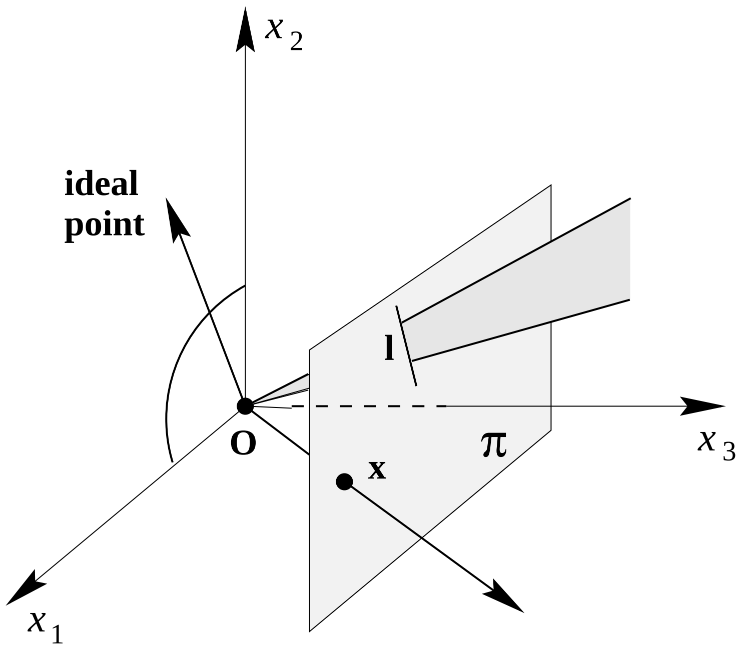
Proof:

for any ideal point $x = (x_1, x_2, 0)^T$: $x^T l_\infty = 0$.
for any finite (real-valued) point $x = (x_1, x_2, 1)$: $x^T l_\infty = 1 \neq 0$.

Furthermore:

- This is the only line in $\mathbb{P}^2$ **not** corresponding to an Euclidean line.
- Two parallel lines meet at the line at infinity.

# A model for the projective plane




- points correspond to rays (lines through the origin)
- lines correspond to planes through the origin. [HZ04, p. 29]

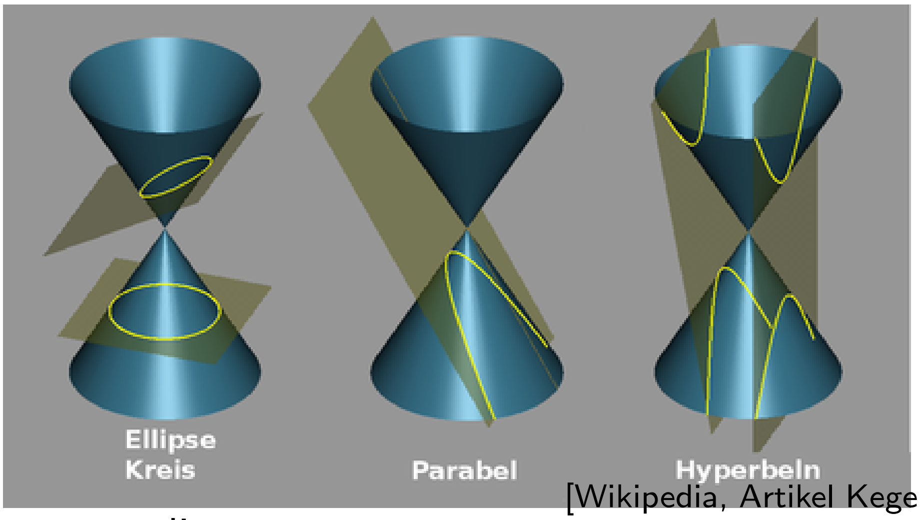




# Conics

- A **conic section** (or just **conic**) is a curve one gets as intersection of a cone and a plane
  - ellipsis, parabola, hyperbola
- Corresponds to a curve of degree 2:
  Heterogeneous coordinates:

  $$a \in \mathbb{R}^6 : \mathbf{C}_a := \{x \in \mathbb{R}^2 \mid a_1 x_1^2 + a_2 x_1 x_2 + a_3 x_2^2 + a_4 x_1 + a_5 x_2 + a_6 = 0\}$$




[Wikipedia, Artikel Kegelschnitt]

Homogeneous coordinates:

# A conic joining 5 points

- Let $x^1, \ldots, x^5 \in \mathbb{P}^2$ be 5 points
  - in general position (i.e., never more than 2 on the same line)
- Conic parameters $a$ have to fulfil the following system of linear equations:

$$\begin{pmatrix} x_1^1x_1^1 & x_1^1x_2^1 & x_2^1x_2^1 & x_1^1x_3^1 & x_2^1x_3^1 & x_3^1x_3^1 \\ x_1^2x_1^2 & x_1^2x_2^2 & x_2^2x_2^2 & x_1^2x_3^2 & x_2^2x_3^2 & x_3^2x_3^2 \\ x_1^3x_1^3 & x_1^3x_2^3 & x_2^3x_2^3 & x_1^3x_3^3 & x_2^3x_3^3 & x_3^3x_3^3 \\ x_1^4x_1^4 & x_1^4x_2^4 & x_2^4x_2^4 & x_1^4x_3^4 & x_2^4x_3^4 & x_3^4x_3^4 \\ x_1^5x_1^5 & x_1^5x_2^5 & x_2^5x_2^5 & x_1^5x_3^5 & x_2^5x_3^4 & x_3^5x_3^5 \end{pmatrix} a = 0$$





# Degenerate Conics

Conic $C$ **degenerate**: $C$ does not have full rank.

Example: two lines $C := ab^T + ba^T$ (rank 2).

- contains lines $a$ and $b$.
  proof: for points $x$ on line $a$: $x^T a = 0$.
  $\rightsquigarrow$ $x$ also on $C$: $x^T C x = x^T a b^T x + x^T b a^T x = 0$.

# Conic tangent lines

The tangent line to a conic $C$ at a point $x$ is $Cx$.

Proof:

x lies on $Cx$: $x^T Cx = 0$.
If there is another common point $y$: $y^T Cy = 0$ and $y^T Cx = 0$.
$\rightsquigarrow x + \alpha y$ is common for all $\alpha$, i.e., the whole line.
$\rightsquigarrow C$ is degenerate (or there is no such $y$).





# Outline

# Projectivity

A map $h : \mathbb{P}^2 \rightarrow \mathbb{P}^2$ is called **projectivity**, if

1. it is invertible and
2. it preserves lines,
   i.e., whenever $x, y, z$ are on a line, so are $h(x), h(y), h(z)$.

Equivalently, $h(x) := Hx$ for a non-singular $H \in \mathbb{P}^{3\times 3}$.

Proof:
Any map $h(x) := Hx$ is a projectivity:
Let $x$ be a point on line $a$: $a^T x = 0$.
Then point $Hx$ is on line $H^{-T}a$: $(H^{-1}a)^T Hx = a^T H^{-1} Hx = a^T x = 0$.

Any projectivity $h$ is of type $h(x) = Hx$: more difficult to show.

Note: $H^{-T} := (H^{-1})^T$.





# Transformation of Lines and Conics

The image of a line $a$ under projectivity $H$ is the line $H^{-T}a$:

$$H(l_a) = l_{H^{-T}a}$$

Proof:
Let $x$ be a point on line $a$: $a^T x = 0$.
Then point $Hx$ is on line $H^{-1}a$: $(H^{-T}a)^T Hx = a^T H^{-1} Hx = a^T x = 0$.

The image of a conic $C$ under projectivity $H$ is the conic $H^{-T}CH^{-1}$:

$$H(\mathbf{C}_C) = \mathbf{C}_{H^{-T}CH^{-1}}$$

Proof:
Let $x$ be a point on conic $C$: $x^T Cx = 0$.
Then point $Hx$ is on conic $H^{-T}CH^{-1}$: $x^T H^T H^{-T} CH^{-1} H^{-1} x = 0$

# A Hierarchy of Transformations

- The projective transformations form a group (**projective linear group**:
$$\mathrm{PL}_n := \mathrm{GL}_n / \equiv = \{H \in \mathbb{P}^{3\times 3} \mid H \text{ invertible}\}$$
- There are several subgroups:
  - **affine group**: last row is $(0, 0, 1)$
  - **Euclidean group**: additionally $H_{1:2,1:2}$ orthogonal
  - **oriented Euclidean group**: additionally $\det H = 1$
- These subgroups can be described two ways:
  - structurally (as above)
  - by **invariants**: objects or sets of objects mapped to themselves





# Isometries

$$\begin{pmatrix} x'_1 \\ x'_2 \\ 1 \end{pmatrix} = \begin{pmatrix} \epsilon \cos\theta & -\sin\theta & t_1 \\ \epsilon \sin\theta & \cos\theta & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} x$$

- **rotation matrix** $R$: $R^T R = R R^T = I$
- **translation vector** $t$.
- **orientation preserving** if $\epsilon = +1$ (equivalent to $\det R = +1$) ($\epsilon \in \{+1, -1\}$)

Invariants:

- length, angle, area
- line at infinity $l_\infty$

# Similarity Transformations

$$\begin{pmatrix} x'_1 \\ x'_2 \\ 1 \end{pmatrix} = \begin{pmatrix} s\cos\theta & -s\sin\theta & t_1 \\ s\sin\theta & s\cos\theta & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} sR & t \\ 0^T & 1 \end{pmatrix} x$$

- **isotropic scaling** $s$.

Invariants:

- angle
- ratio of lengths, ratio of areas
- line at infinity $l_\infty$





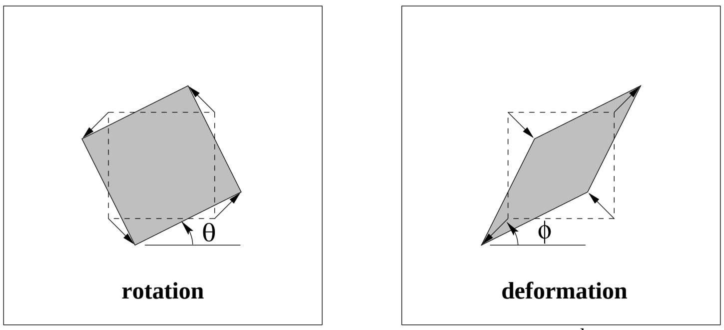
# Affine Transformations

$$\begin{pmatrix} x'_1 \\ x'_2 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & t_1 \\ a_{2,1} & a_{2,2} & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & t \\ 0^T & 1 \end{pmatrix} x$$

- $A$ non-singular, decompose via SVD:

$$A = R(\theta)R(-\phi) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} R(\phi)$$

- **non-isotropic scaling** with axis $\phi$




[HZ04, p. 40]

# Projective Transformations

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & t_1 \\ a_{2,1} & a_{2,2} & t_2 \\ v_1 & v_2 & v_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} A & t \\ v^T & v_3 \end{pmatrix} x$$

- **$v$ moves the line at infinity $l_\infty$**

Invariants:

- ratio of ratios of lengths of parallel line segments (**cross ratio**)





# Similary, Affine & Projective Transformations / Example

a

b

c

| | a) similarity | b) affine | c) projective |
|---|---|---|---|
| circles | circles | ellipsis | conic |
| squares | squares | diamond | quadrangle |
| parallel lines | parallel | parallel | converging |
| orthogonal linnes | orthogonal | non-orthogonal | non-orthogonal |

[HZ04, p. 37]

# Projective Transformations / Decomposition

$$\begin{pmatrix} A & t \\ v^T & v_3 \end{pmatrix} = \begin{pmatrix} sR & t \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} I & 0 \\ v^T & v_3 \end{pmatrix}$$

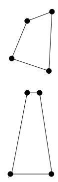
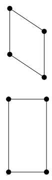
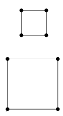
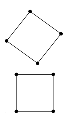
$$A = sRK + tv^T$$

- $K$ upper triangular matrix with $\det K = 1$
- valid for $v_3 \neq 0$
- unique if $s$ is chosen $s > 0$





# Summary of Projective Transformations

| Group | Matrix | Distortion | Invariant properties |
|---|---|---|---|
| Projective<br>8 dof | $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ |  | Concurrency, collinearity, **order of contact**: intersection (1 pt contact); tangency (2 pt contact); inflections (3 pt contact with line); tangent discontinuities and cusps. cross ratio (ratio of ratio of lengths). |
| Affine<br>6 dof | $\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ |  | Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines (e.g. midpoints), linear combinations of vectors (e.g. centroids). The line at infinity, $\mathbf{l}_\infty$. |
| Similarity<br>4 dof | $\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ |  | Ratio of lengths, angle. The circular points, **I**, **J** (see section 2.7.3). |
| Euclidean<br>3 dof | $\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ |  | Length, area |

[HZ04, p. 44]

# Outline







# Recovery of Affine and Metric Properties

Decomposition of general projective transformation:

$$\begin{pmatrix} A & t \\ v^T & v_3 \end{pmatrix} = \begin{pmatrix} sR & t \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} I & 0 \\ v^T & v_3 \end{pmatrix}$$

1. undo proper projective transformation (**affine rectification**):
   - then original and image differ only by an affine transformation
   - $\rightsquigarrow$ measure **affine properties** of the original in the image (= properties invariant under affine transformations)
     - parallel lines, ratio of lengths on parallel lines

2. undo proper affine transformation (**metric rectification**):
   - then original and image differ only by a similarity transformation
   - $\rightsquigarrow$ measure **metric properties** of the original in the image (= properties invariant under similarity transformations)
     - angles, ratio of lengths

# Recovery of Affine Properties

Undo proper projective transformation:

$$\begin{pmatrix} I & 0 \\ v^T & v_3 \end{pmatrix} : \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} x_1 \\ x_2 \\ v_1 x_1 + v_2 x_2 \end{pmatrix}$$

$$l_\infty := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} -v/v_3 \\ 1/v_3 \end{pmatrix} = \frac{1}{v_3} \begin{pmatrix} v_1 \\ v_2 \\ 1 \end{pmatrix}$$

- maps line at infinity to finite line $(v_1, v_2, 1)^T$
- to undo:
  - locate image $(v_1, v_2, 1)^T$ of line at infinity
  - undo by applying the inverse $H^{-1} = \begin{pmatrix} I & 0 \\ -v^T/v_3 & 1/v_3 \end{pmatrix}$

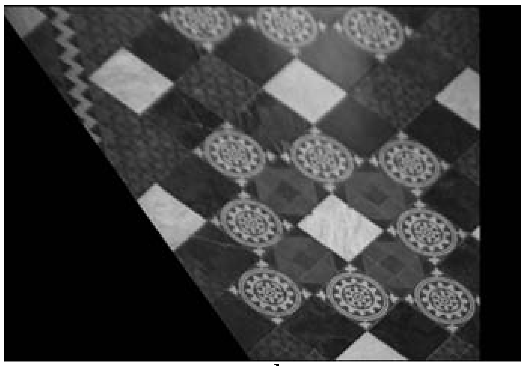
Note: Lines transform by $H^{-T}$: $\begin{pmatrix} I & 0 \\ v^T & v_3 \end{pmatrix}^{-T} = \begin{pmatrix} I & -v/v_3 \\ 0 & 1/v_3 \end{pmatrix}$.



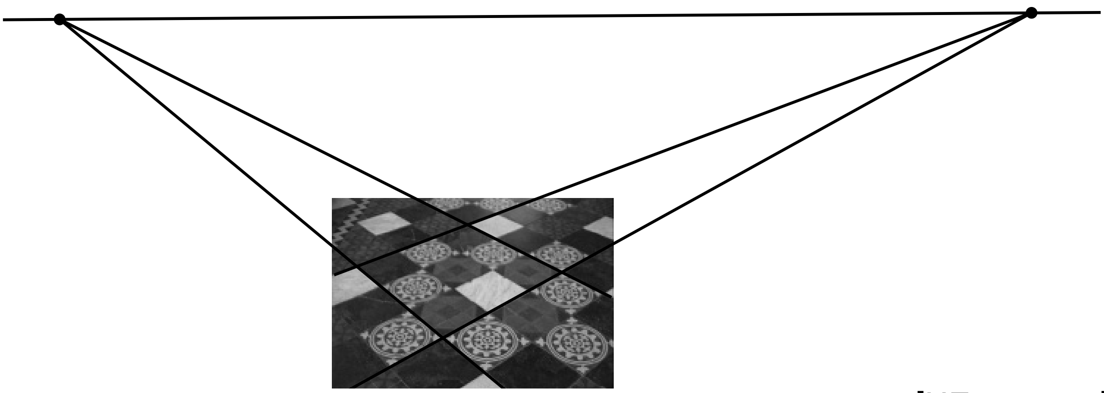


# Recovery of Affine Properties / Example

a



b



[HZ04, p. 50]

Now we can measure area ratios !

# Recovery of Affine Properties / Algorithm

1: **procedure** RECTIFY-AFFINE-TWO-PARALLELS($a^1, a^2, b^1, b^2 \in \mathbb{P}^2$)
2: $s^1 := a^1 \times a^2$ ▷ compute intersection of parallels $a^1, a^2$
3: $s^2 := b^1 \times b^2$ ▷ compute intersection of parallels $b^1, b^2$
4: $l_\infty := s^1 \times s^2$ ▷ compute image of line at infinity
5: $H^{-1} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -l_{\infty,1}/l_{\infty,3} & -l_{\infty,2}/l_{\infty,3} & 1/l_{\infty,3} \end{pmatrix}$ ▷ compute inverse
6: **return** $H^{-1}$





# Outline

# Circular Points

A conic

$$C := \begin{pmatrix} a_1 & a_2/2 & a_4/2 \\ a_2/2 & a_3 & a_5/2 \\ a_4/2 & a_5/2 & a_6 \end{pmatrix} = \begin{pmatrix} a_1 & 0 & a_4/2 \\ 0 & a_1 & a_5/2 \\ a_4/2 & a_5/2 & a_6 \end{pmatrix}$$

is a circle if $a_1 = a_3$ and $a_2 = 0$.

Ideal points $x = (x_1, x_2, 0)^T$ on a circle:

$$x^T C x = a_1 x_1^2 + a_1 x_2^2 = 0$$

are exactly the **circular points**:

$$I := \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad J := \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$





# Line Conics

$C \in \mathrm{Sym}(\mathbb{P}^{3\times 3})$ defines a **point conic** via

$$\mathbf{C}_C := \{x \in \mathbb{P}^2 \mid x^T C x = 0\}$$

It also can be used to define a **line conic** / **dual conic**:

$$\mathbf{C}_C^* := \{a \in \mathbb{P}^2 \mid a^T C a = 0\}$$

(where $a$ denotes a line)

# Adjugate of a Matrix

For a square matrix $A \in \mathbb{R}^{n \times n}$,

$$A^* \in \mathbb{R}^{n \times n} \text{ with } A^*_{i,j} := (-1)^{i+j} \det A_{-j,-i}$$

is called its **adjugate** $A^*$.

It holds:

- for any $A$: $A^* A = AA^* = (\det A) I$
- $A^*$ is continuous in $A$.
- if $A$ is invertible, the adjoint is the scaled inverse: $A^* = (\det A) A^{-1}$
- if $A$ is not invertible, the adjoint nullifies $A$: $A^* A = AA^* = 0$
- the adjugate is the transposed of the cofactor matrix.

Note: $A_{-j,-i}$ denotes the matrix $A$ with row $j$ and column $i$ removed.
The adjugate is also called **adjoint**.



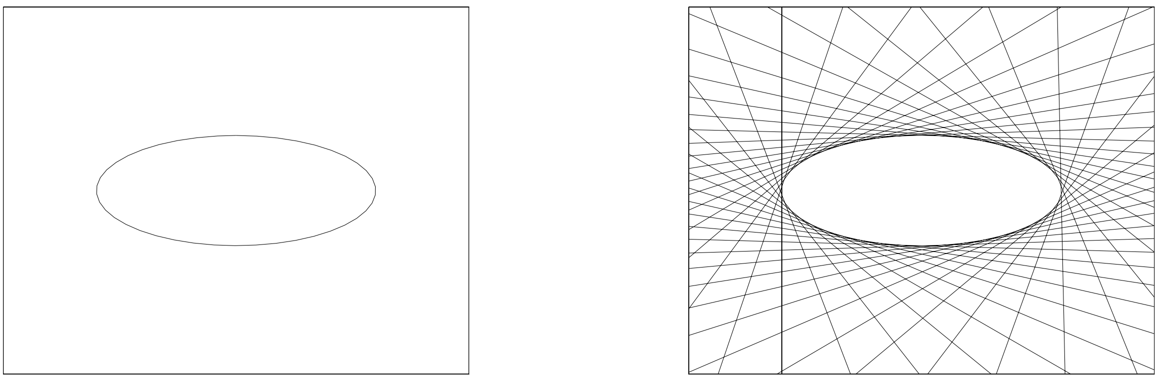


# Dual Conic

For any point conic $C \in \text{Sym}(\mathbb{P}^{3\times 3})$, the set of tangent lines

- forms a line conic,
- parametrized by the adjugate $C^*$:

$$\{a \in \mathbb{P}^2 \mid a \text{ tangent to } C\} = \mathbf{C}^*_{C^*}$$



a b

[HZ04, p. 32]

# Dual Conic to the Circular Points

**Dual conic to the circular points** (degenerate):

$$C_\infty^* := IJ^T + JI^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- contains exactly all lines through the circular points $I$ or $J$.
- transforms as $HC^*H^T$: $H(\mathbf{C}^*_{C^*}) = \mathbf{C}^*_{HC^*H^T}$.
- fixed under projectivity $H$ iff $H$ is a similarity.
- 4 dof (general $C$ has 5, minus 1 due to $\det C = 0$)
- $l_\infty$ is the null vector of $C_\infty^*$.





# Angels in the Projective Plane

Angels are defined as:

$$\cos\theta(a,b) := \frac{a^T C_\infty^* b}{\sqrt{(a^T C_\infty^* a)\,(b^T C_\infty^* b)}}, \quad a, b \in \mathbb{P}^2$$

- for the canonical $C_\infty^*$, conincides with the Euclidean definition:

$$\cos\theta(a,b) := \frac{a^T b}{\sqrt{(a^T a)\,(b^T b)}}, \quad a, b \in \mathbb{R}^2$$

- stays invariant under projective transformation:

$$a' = H^{-T} a, \quad b' = H^{-T} b, \quad C_\infty^{*}{}' = H C_\infty^* H^T$$
$$a'^T C_\infty^{*}{}' b' = a^T H^{-1} H C_\infty^* H^T H^{-T} b = a^T C_\infty^* b$$

# Outline





Computer Vision 6. Recovery of Metric Properties from Images

# Recovery of Metric Properties

- assume there is no pure projective transformation (i.e., affine rectification already done).
- need only to find pure affine transformation:

$$H_a := \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix}, \quad \text{with } K \text{ upper triangular}$$

- under $H_a$ we get $C_\infty^*{}'$ as

$$C_\infty^*{}' := H_a C_\infty^* H_a^T = \begin{pmatrix} KK^T & 0 \\ 0^T & 0 \end{pmatrix}$$

1. find symmetric matrix $S := KK^T$
2. find $K$ via Cholesky decomposition of $S$

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# Recovery of Metric Properties (2/2)

- for two lines $a', b'$ that are orthogonal in the original:

$$\begin{aligned} 0 &= a'^T C_\infty^{*\prime} b' = {a'_{1:2}}^T S b_{1:2} \\ &= a'_1 S_{1,1} b'_1 + a'_1 S_{1,2} b'_2 + a'_2 S_{2,1} b'_1 + a'_2 S_{2,2} b'_2 \\ &= a'_1 b'_1 S_{1,1} + (a'_1 b'_2 + a'_2 b'_1) S_{1,2} + a'_2 b'_2 S_{2,2} \\ &= (a'_1 b'_1, a'_1 b'_2 + a'_2 b'_1, a'_2 b'_2)(S_{1,1}, S_{1,2}, S_{2,2})^T \end{aligned}$$

  we get 1 linear constraint in $s := (S_{1,1}, S_{1,2}, S_{2,2})^T$. .
- for two pairs of lines that are orthogonal in the original we get 2 linear constraints for 3 variables

$$\begin{pmatrix} a'_1 b'_1 & a'_1 b'_2 + a'_2 b'_1 & a'_2 b'_2 \\ c'_1 d'_1 & c'_1 d'_2 + c'_2 d'_1 & c'_2 d'_2 \end{pmatrix} s$$

  where $s \neq 0$ has to be identified only up to a factor.





# Recovery of Metric Properties / Algorithm

1: **procedure** $\textsc{Rectify-Metric-Two-Orthogonals}(a^1, a^2, b^1, b^2 \in \mathbb{P}^2)$

2: $\quad A := \begin{pmatrix} a_1^1 a_1^2 & a_1^1 a_2^2 + a_2^1 a_1^2 & a_2^1 a_2^2 \\ b_1^1 b_1^2 & b_1^1 b_2^2 + b_2^1 b_1^2 & b_2^1 b_2^2 \end{pmatrix}$

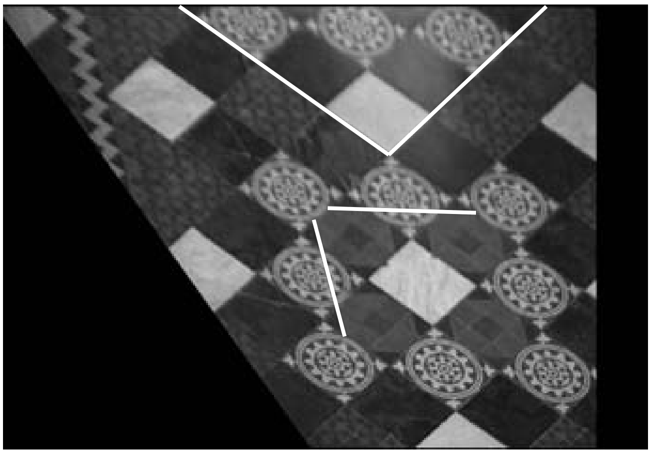
3: $\quad$ find $s \neq 0 : As = 0$ $\qquad \triangleright$ find $C_\infty^* := \begin{pmatrix} s_1 & s_2 & 0 \\ s_2 & s_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

4: $\quad K :=$ cholesky$(\begin{pmatrix} s_1 & s_2 \\ s_2 & s_3 \end{pmatrix})$ $\qquad \triangleright$ find $H := \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix}$

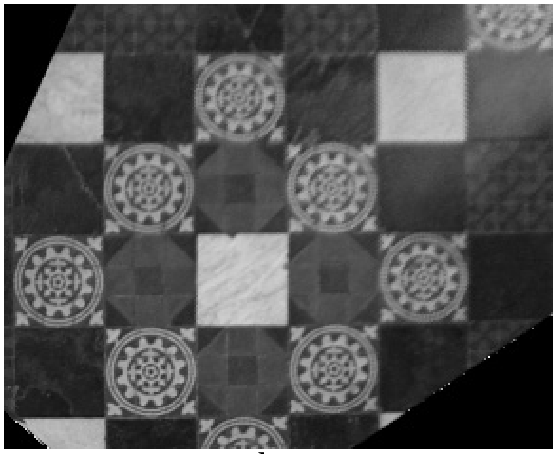
5: $\quad H^{-1} := \begin{pmatrix} 1/K_{1,1} & -1/(K_{1,2}K_{2,2}) & 0 \\ 0 & 1/K_{2,2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\qquad \triangleright$ compute inverse

6: $\quad$ **return** $H^{-1}$

# Recovery of Metric Properties / Example



a



b

a) affine rectified image

b) metric rectified image

Now we can measure angles and length ratios !

[HZ04, p. 57]





# Outline

# Exercises and Tutorials

- There will be a weekly sheet with 4 exercises
  handed out **each Tuesday** in the lecture.
  1st sheet will be handed out Thu. 23.4. in the tutorial.

- Solutions to the exercises can be
  submitted until **next Tuesday noon**
  1st sheet is due Tue. 28.4.

- Exercises will be corrected.

- Tutorials **each Thursday 2pm–4pm**,
  1st tutorial at Thur. 23.4.

- Successful participation in the tutorial gives up to 10% bonus points for the exam.





# Exam and Credit Points

- There will be a written exam at end of term
  (2h, 4 problems).

- The course gives 6 ECTS (2+2 SWS).

- The course can be used in
  - IMIT MSc. / Informatik / Gebiet KI & ML
  - Wirtschaftsinformatik MSc / Informatik / Gebiet KI & ML
  - as well as in both BSc programs.

# Some Text Books

- Simon J. D. Prince (2012):
  *Computer Vision: Models, Learning, and Inference,*
  Cambridge University Press.

- Richard Szeliski (2011):
  *Computer Vision, Algorithms and Applications*,
  Springer.

- David A. Forsyth, Jean Ponce ($^2$2012, 2007):
  *Computer Vision, A Modern Approach,*
  Prentice Hall.

- Richard Hartley, Andrew Zisserman (2004):
  *Multiple View Geometry in Computer Vision*,
  Cambridge University Press.





# Some First Computer Vision Software

- Open Computer Vision Library (OpenCV)
  - C++ library
  - has wrappers for Python & Octave
  - originally developed by Intel
  - v3.0 beta, 11/2014; `http://opencv.org`

Public data sets:

- ...

# Summary (1/3)

- The **projective plane** $\mathbb{P}^2$ is an extension of the Euclidean plane with **ideal points**.
- Points and lines in $\mathbb{P}^2$ are parametrized by **homogenuous coordinates**.
- Each two parallels intersect in an ideal point, all ideal points form the **line at infinity** $l_\infty$.
- Each circle contains two ideal points, the **circular points**, all lines through the circular points form the **dual conic to the circular points** $C^*_\infty$.
- **Conics** are curves of order 2 (hyperbolas, parabolas, ellipsis), parametrized by a symmetric matrix $C$ containing all points $x$ with $x^T Cx = 0$.





# Summary (2/3)

- **Projectivities** $H$ are invertibles mappings of $\mathbb{P}^2$ onto $\mathbb{P}^2$ that preserve lines.
- Lines $a$ transform via $H^{-T}a$, conics $C$ via $H^{-T}CH^{-1}$.
- There exist several subgroups of the group of projectivities:
  - **Isometries** **rotate** and **translate** figures.
    - preserving lengths
  - **Similarities** additionally **(isotropic) scale** figures.
    - preserving ratio of lengths, angle
  - **Affine transforms** additionally **non-isotropic scale** figures.
    - preserving ratio of lengths on parallel lines, parallel lines
  - **Projectivities** additionally **move the line at infinity**.
    - preserving cross ratio
- Any projectivity can be decomposed into a chain of an pure projectivie, a pure affine transform and a similarity.

# Summary (3/3)

- Images distorted by an projective transformation can be **rectified** (i.e., undoes the projective transformation).
- **Affine rectification**
  - undoes a proper projective transformation
  - moves the **line at infinity** back to its **canonical position**.
  - allows to measure **affine properties**:
    - ratio of lengths on parallel lines, parallel lines
  - requires, e.g., **two pairs of parallel lines**.
- **Metric rectification**
  - undoes a proper affine transformation
  - moves the **dual conic to the circular points** back to its canonical position.
  - allows to measure **metric properties**:
    - angles, ratio of lengths
  - requires, e.g., two pairs of orthogonal lines.





# Further Readings

- [HZ04, ch. 1 and 2].

# References

Richard Hartley and Andrew Zisserman.
*Multiple view geometry in computer vision*.
Cambridge university press, 2004.