Computer Vision

1. Projective Geometry in 2D

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Outline

1. Very Brief Introduction

2. The Projective Plane

3. Projective Transformations

4. Recovery of Affine and Metric Properties from Images

5. Organizational Stuff

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1. Very Brief Introduction

2. The Projective Plane

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5. Organizational Stuff

Topics of the Lecture

1. Simultaneous Localization and Mapping from Video (Visual SLAM)
2. Image Classification and Description

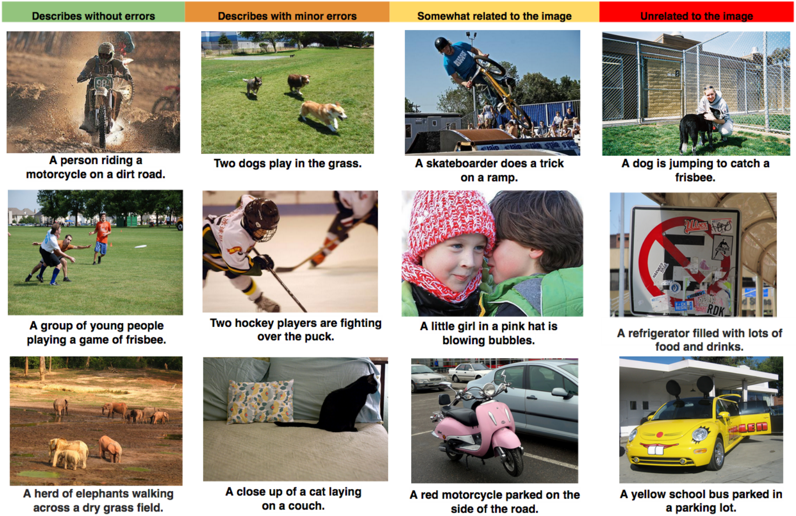
Simultaneous Localization and Mapping

[source https://www.youtube.com/watch?v=bD0nn0-4Nq8]

Simultaneous Localization and Mapping from Video

- SLAM usually employs laser range scanners (lidars).
- **Visual SLAM**: use video sensors (cameras).
- main parts required:
  1. Projective Geometry
  2. Point Correspondences
  3. Estimating Camera Positions (Localization)
  4. Triangulation (Mapping)

Image Classification and Description



[source: http://googleresearch.blogspot.de/2014/11/a-picture-is-worth-thousand-coherent.html]

Outline

Motivation

In Euclidean (planar) geometry, there are many exceptions, e.g.,

- most two lines intersect in exactly one point.
- but some two lines do not intersect.
  - parallel lines

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- but some two lines do not intersect.
  - parallel lines

Idea:

- add **ideal points**, one for each set of parallel lines / direction
- define these points as intersection of any two parallel lines
- now any two lines intersect in exactly one point
  - either in a finite or in an ideal point

Homogeneous Coordinates: Points

Inhomogeneous coordinates:

$$x \in \mathbb{R}^2$$

Homogeneous coordinates:

$$x \in \mathbb{P}^2 := \mathbb{R}^3 / \equiv$$

$$x \equiv y :\Longleftrightarrow \exists s \in \mathbb{R} \setminus \{0\} : sx = y, \quad x, y \in \mathbb{R}^3$$

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Example:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \equiv \begin{pmatrix} 4 \\ 8 \\ 12 \end{pmatrix} \text{ represent the same point in } \mathbb{P}^2$$

$$\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \text{ represent a different point in } \mathbb{P}^2$$

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finite points: $\begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} =: \iota(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix})$

ideal points: $\begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$

Homogeneous Coordinates: Lines

Inhomogeneous coordinates:

$$a \in \mathbb{R}^3 : \ell_a := \{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid a_1 x_1 + a_2 x_2 + a_3 = 0 \}$$

- $a_1 \neq 0$ or $a_2 \neq 0$ (or both $a_1, a_2 \neq 0$).
- $sa = (sa_1, sa_2, sa_3)^T$ encodes the same line as a (any $s \in \mathbb{R}, s \neq 0$).

Note: $\kappa : \mathbb{R}^3 \to \mathbb{P}^2, a \mapsto [a] := \{a' \in \mathbb{R}^3 \mid a' \equiv a\}$.

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Homogeneous coordinates:

$$a \in \mathbb{P}^2 : \ell_a := \{ x \in \mathbb{P}^2 \mid a^T x = a_1 x_1 + a_2 x_2 + a_3 x_3 = 0 \}$$

- contains all finite points of $a' \in \kappa^{-1}(a)$: $\ell_{\kappa(a')} \supsetneqq \iota(\ell_{a'})$
- and the ideal point $(a_2, -a_1, 0)^T$.
  - intersection of parallel lines (same a_1, a_2, different a_3)

Note: $\kappa : \mathbb{R}^3 \to \mathbb{P}^2, a \mapsto [a] := \{ a' \in \mathbb{R}^3 \mid a' \equiv a \}$.

A point on a line

A point x lies on line a iff $x^T a = 0$.

Intersection of two lines

Lines a and b intersect in $a \times b := \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ -a_1 b_3 + a_3 b_1 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

Proof:

$$a^T(a \times b) = a_1 a_2 b_3 - a_1 a_3 b_2 - a_2 a_1 b_3 + a_2 a_3 b_1 + a_3 a_1 b_2 - a_3 a_2 b_1 = 0$$
$$b^T(a \times b) = \ldots = 0$$

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Example:

$$x = 1 : a = (-1, 0, 1)^T$$
$$y = 1 : b = (0, -1, 1)^T$$
$$a \times b = (1, 1, 1)^T$$

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$$b^T(a \times b) = \ldots = 0$$

Esp. for parallel lines: $b_1 = a_1, b_2 = a_2, b_3 \neq a_3$:

$$a \times b \equiv \begin{pmatrix} a_2 \\ -a_1 \\ 0 \end{pmatrix}$$

Line joining points

The line through x and y is $x \times y$.

Proof: exactly the same as previous slide.

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Example:

$$\begin{aligned} x =& (-1, 0, 1)^T \\ y =& (0, -1, 1)^T \\ x \times y =& (1, 1, 1)^T \end{aligned}$$

Line at infinity

All ideal points form a line:

$$l_\infty := (0, 0, 1)^T \quad \textbf{line at infinity}$$

Proof:

for any ideal point $x = (x_1, x_2, 0)^T$: $x^T l_\infty = 0$.

for any finite (real-valued) point $x = (x_1, x_2, 1)$: $x^T l_\infty = 1 \neq 0$.

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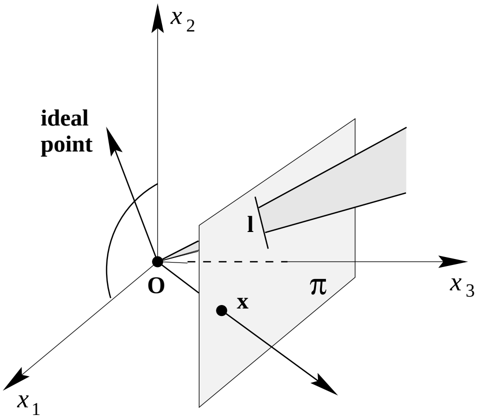
for any finite (real-valued) point $x = (x_1, x_2, 1)$: $x^T l_\infty = 1 \neq 0$.

Furthermore:

- This is the only line in $\mathbb{P}^2$ **not** corresponding to an Euclidean line.
- Two parallel lines meet at the line at infinity.

A model for the projective plane



- points correspond to rays (lines through the origin)
- lines correspond to planes through the origin.

[HZ04, p. 29]

Conics

- A **conic section** (or just **conic**) is a curve one gets as intersection of a cone and a plane
  - hyperbola, parabola, ellipsis
- Corresponds to a curve of degree 2:
  Heterogeneous coordinates:

$$a \in \mathbb{R}^6 : \mathbf{C}_a := \{x \in \mathbb{R}^2 \mid a_1 x_1^2 + a_2 x_1 x_2 + a_3 x_2^2 + a_4 x_1 + a_5 x_2 + a_6 = 0\}$$

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  Homogeneous coordinates:

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$$= \{x \in \mathbb{P}^2 \mid x^T C x = 0\}, C := \begin{pmatrix} a_1 & a_2/2 & a_4/2 \\ a_2/2 & a_3 & a_5/2 \\ a_4/2 & a_5/2 & a_6 \end{pmatrix}$$

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Homogeneous coordinates:

$$C \in \mathrm{Sym}(\mathbb{P}^{3\times 3}) : \mathbf{C}_C := \{x \in \mathbb{P}^2 \mid x^T C x = 0\}$$

A conic joining 5 points

- Let $x^1, \ldots, x^5 \in \mathbb{P}^2$ be 5 points
  - in general position (i.e., never more than 2 on the same line)
- Conic parameters a have to fulfil the following system of linear equations:

$$\begin{pmatrix} x_1^1 x_1^1 & x_1^1 x_2^1 & x_2^1 x_2^1 & x_1^1 x_3^1 & x_2^1 x_3^1 & x_3^1 x_3^1 \\ x_1^2 x_1^2 & x_1^2 x_2^2 & x_2^2 x_2^2 & x_1^2 x_3^2 & x_2^2 x_3^2 & x_3^2 x_3^2 \\ x_1^3 x_1^3 & x_1^3 x_2^3 & x_2^3 x_2^3 & x_1^3 x_3^3 & x_2^3 x_3^3 & x_3^3 x_3^3 \\ x_1^4 x_1^4 & x_1^4 x_2^4 & x_2^4 x_2^4 & x_1^4 x_3^4 & x_2^4 x_3^4 & x_3^4 x_3^4 \\ x_1^5 x_1^5 & x_1^5 x_2^5 & x_2^5 x_2^5 & x_1^5 x_3^5 & x_2^5 x_3^4 & x_3^5 x_3^5 \end{pmatrix} a = 0$$

Degenerate Conics

Conic C **degenerate**: C does not have full rank.

Example: two lines $C := ab^T + ba^T$ (rank 2).

- contains lines a and b.
  proof: for points x on line a: $x^T a = 0$.
  $\rightsquigarrow$ x also on C: $x^T Cx = x^T ab^T x + x^T ba^T x = 0$.

Conic tangent lines

The tangent line to a conic C at a point x is Cx.

Proof:

x lies on Cx: $x^T Cx = 0$.

If there is another common point y: $y^T Cy = 0$ and $y^T Cx = 0$.

$\rightsquigarrow$ $x + \alpha y$ is common for all α, i.e., the whole line.

$\rightsquigarrow$ C is degenerate (or there is no such y).

Outline

Projectivity

A map $h : \mathbb{P}^2 \rightarrow \mathbb{P}^2$ is called **projectivity**, if

1. it is invertible and
2. it preserves lines,
   i.e., whenever x, y, z are on a line, so are $h(x), h(y), h(z)$.

Equivalently, $h(x) := Hx$ for a non-singular $H \in \mathbb{P}^{3\times 3}$.

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Equivalently, $h(x) := Hx$ for a non-singular $H \in \mathbb{P}^{3\times 3}$.

Proof:
Any map $h(x) := Hx$ is a projectivity:
Let x be a point on line a: $a^T x = 0$.
Then point Hx is on line $H^{-1}a$: $(H^{-1}a)^T Hx = a^T H^{-1} Hx = a^T x = 0$.

Any projectivity h is of type $h(x) = Hx$: more difficult to show.

Transformation of Lines and Conics

The image of a line a under projectivity H is the line $H^{-1}a$:

$$H(l_a) = l_{H^{-1}a}$$

Proof:

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The image of a conic C under projectivity H is the conic $H^{-T} C H^{-1}$:

$$H(\mathbf{C}_C) = \mathbf{C}_{H^{-T} C H^{-1}}$$

Proof:

Let x be a point on conic C: $x^T C x = 0$.
Then point Hx is on conic $H^{-T} C H^{-1}$: $x^T H^T H^{-T} C H^{-1} H^{-1} x = 0$

A Hierarchy of Transformations

- The projective transformations form a group (**projective linear group**:

$$\text{PL}_n := \text{GL}_n / \equiv = \{H \in \mathbb{P}^{3\times 3} \mid H \text{ invertible}\}$$

A Hierarchy of Transformations

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- There are several subgroups:
  - **affine group**: last row is $(0, 0, 1)$
  - **Euclidean group**: additionally $H_{1:2,1:2}$ orthogonal
  - **oriented Euclidean group**: additionally $\det H = 1$

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- These subgroups can be described two ways:
  - structurally (as above)
  - by **invariants**: objects or sets of objects mapped to themselves

Isometries

$$\begin{pmatrix} x'_1 \\ x'_2 \\ 1 \end{pmatrix} = \begin{pmatrix} \epsilon \cos\theta & -\sin\theta & t_1 \\ \epsilon \sin\theta & \cos\theta & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} x$$

- **rotation matrix** R: $R^T R = R R^T = I$
- **translation vector** t.
- **orientation preserving** if $\epsilon = +1$ (equivalent to $\det R = +1$) ($\epsilon \in \{+1, -1\}$)

Isometries

$$\begin{pmatrix} x_1' \\ x_2' \\ 1 \end{pmatrix} = \begin{pmatrix} \epsilon \cos\theta & -\sin\theta & t_1 \\ \epsilon \sin\theta & \cos\theta & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} x$$

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Invariants:

- length, angle, area
- line at infinity l_∞

Similarity Transformations

$$\begin{pmatrix} x_1' \\ x_2' \\ 1 \end{pmatrix} = \begin{pmatrix} s\cos\theta & -s\sin\theta & t_1 \\ s\sin\theta & s\cos\theta & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} sR & t \\ 0^T & 1 \end{pmatrix} x$$

- **isotropic scaling** s.

Similarity Transformations

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- **isotropic scaling** s.

Invariants:

- angle
- ratio of lengths, ratio of areas
- line at infinity l_∞

Affine Transformations

$$\begin{pmatrix} x'_1 \\ x'_2 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & t_1 \\ a_{2,1} & a_{2,2} & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & t \\ 0^T & 1 \end{pmatrix} x$$

- A non-singular

Affine Transformations

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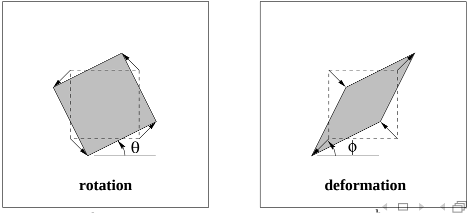
- A non-singular, decompose via SVD:

$$A = R(\theta)R(-\phi)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} R(\phi)$$

- **non-isotropic scaling** with axis ϕ



[HZ04, p. 40]

Affine Transformations

$$\begin{pmatrix} x_1' \\ x_2' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & t_1 \\ a_{2,1} & a_{2,2} & t_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & t \\ 0^T & 1 \end{pmatrix} x$$

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Invariants:

- parallel lines
- ratio of lengths of parallel line segments
- ratio of areas
- line at infinity l_∞

Projective Transformations

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & t_1 \\ a_{2,1} & a_{2,2} & t_2 \\ v_1 & v_2 & v_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} A & t \\ v^T & 1 \end{pmatrix} x$$

- **v moves the line at infinity l_∞**

Projective Transformations

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- **v moves the line at infinity l_∞**

Invariants:

- ratio of ratios of lengths of parallel line segments (**cross ratio**)

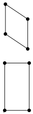
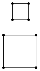
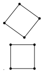
Projective Transformations / Decomposition

$$\begin{pmatrix} A & t \\ v^T & v_3 \end{pmatrix} = \begin{pmatrix} sR & t \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} I & 0 \\ v^T & v_3 \end{pmatrix}$$

$$A = sRK + tv^T$$

- K upper triangular matrix with $\det K = 1$
- valid for $v_3 \neq 0$
- unique if s is chosen $s > 0$

Summary of Projective Transformations

Group	Matrix	Distortion	Invariant properties
Projective 8 dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, **order of contact**: intersection (1 pt contact); tangency (2 pt contact); inflections (3 pt contact with line); tangent discontinuities and cusps. cross ratio (ratio of ratio of lengths).
Affine 6 dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines (e.g. midpoints), linear combinations of vectors (e.g. centroids). The line at infinity, $\mathbf{l}_\infty$.
Similarity 4 dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratio of lengths, angle. The circular points, **I**, **J** (see section 2.7.3).
Euclidean 3 dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Length, area

[HZ04, p. 44]

Outline

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Outline

Exercises and Tutorials

- There will be a weekly sheet with 4 exercises
  handed out **each Tuesday** in the lecture.
  1st sheet will be handed out Thu. 23.4. in the tutorial.

- Solutions to the exercises can be
  submitted until **next Tuesday noon**
  1st sheet is due Tue. 28.4.

- Exercises will be corrected.

- Tutorials **each Thursday 2pm–4pm**,
  1st tutorial at Thur. 23.4.

- Successful participation in the tutorial gives up to 10% bonus points for the exam.

Exam and Credit Points

- There will be a written exam at end of term (2h, 4 problems).
- The course gives 6 ECTS (2+2 SWS).
- The course can be used in
  - IMIT MSc. / Informatik / Gebiet KI & ML
  - Wirtschaftsinformatik MSc / Informatik / Gebiet KI & ML
  - as well as in both BSc programs.

Some Text Books

- Simon J. D. Prince (2012):
  Computer Vision: Models, Learning, and Inference,
  Cambridge University Press.

- Richard Szeliski (2011):
  Computer Vision, Algorithms and Applications,
  Springer.

- David A. Forsyth, Jean Ponce (^{2}2012, 2007):
  Computer Vision, A Modern Approach,
  Prentice Hall.

- Richard Hartley, Andrew Zisserman (2004):
  Multiple View Geometry in Computer Vision,
  Cambridge University Press.

Some First Computer Vision Software

- Open Computer Vision Library (OpenCV)
  - C++ library
  - has wrappers for Python & Octave
  - originally developed by Intel
  - v3.0 beta, 11/2014; `http://opencv.org`

Public data sets:

- ...

Further Readings

- [HZ04, ch. 1 and 2].

References

Richard Hartley and Andrew Zisserman.
Multiple view geometry in computer vision.
Cambridge university press, 2004.