

Outline

1. Smoothing, Image Derivatives, Convolutions
2. Edges, Corners, and Interest Points
3. Image Patch Descriptors
4. Interest Point Matching
5. A Simple Application: Image Stitching

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Computer Vision 1. Smoothing, Image Derivatives, Convolutions

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Smoothing / Blurring / Averaging

- **Smoothing**: Replace each pixel by the weighted average of its surrounding patch:

$$\begin{aligned}
 I_{\text{smooth}}(x, y; w) &:= \sum_{\Delta x, \Delta y} w(-\Delta x, -\Delta y) I(x + \Delta x, y + \Delta y) \\
 &= \sum_{x', y'} w(x - x', y - y') I(x', y')
 \end{aligned}$$

- **padding** with 0 at the image boundaries.
- example: **box kernel**

$$w_{-2:2, -2:2}(\Delta x, \Delta y) := \frac{1}{25} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

- **Gaussian smoothing**: smoothing with a Gaussian kernel.

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Gaussian Kernels

- Precomputed weights: (clipped) Gaussian density values

$$\tilde{w}(\Delta x, \Delta y) := \begin{cases} \mathcal{N}(\sqrt{\Delta x^2 + \Delta y^2}; 0, \sigma^2), & \text{if } |\Delta x| \leq K, |\Delta y| \leq K \\ 0, & \text{else} \end{cases}$$

$$w(\Delta x, \Delta y) := \frac{\tilde{w}(\Delta x, \Delta y)}{\sum_{\Delta x', \Delta y'} \tilde{w}(\Delta x', \Delta y')}$$

- clipped: small support, **window size K** .
- example ($K = 2, \sigma^2 = 1$):

$$w_{-2:2, -2:2} := \begin{pmatrix} 0.003 & 0.013 & 0.022 & 0.013 & 0.003 \\ 0.013 & 0.060 & 0.098 & 0.060 & 0.013 \\ 0.022 & 0.098 & 0.162 & 0.098 & 0.022 \\ 0.013 & 0.060 & 0.098 & 0.060 & 0.013 \\ 0.003 & 0.013 & 0.022 & 0.013 & 0.003 \end{pmatrix}$$

Note: $\mathcal{N}(x; \mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

Blurring / Example

blurred by $G(K = 5, \sigma = 1)$:



blurred by $G(K = 5, \sigma = 10)$:



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Image Derivatives

- **Image Derivative:** How does the intensity values change in x or y direction?

$$I_X(x, y) := I(x, y) - I(x - 1, y)$$

$$I_Y(x, y) := I(x, y) - I(x, y - 1)$$

or symmetric

$$I_X(x, y) := 2I(x, y) - I(x - 1, y) - I(x + 1, y)$$

$$I_Y(x, y) := 2I(x, y) - I(x, y - 1) - I(x, y + 1)$$

Image Derivatives / Example

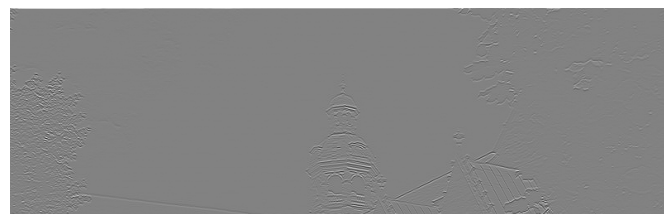
derivative in x-direction:



original (grayscale):



derivative in y-direction:



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Convolutions

- Smoothing, Image Derivatives and further operations such as filtering can be represented by a
 - **convolution**: an image where each pixel (x, y) represents the weighted sum around (x, y) in image I weighted with w :

$$(w * I)(x, y) := \sum_{x', y'} w(x - x', y - y') I(x', y')$$

- Examples:

$$I_{\text{smooth}}$$

$$= w * I$$

$$I_X(x, y) := I(x, y) - I(x - 1, y)$$

$$= \begin{pmatrix} 1 & -1 \end{pmatrix} * I$$

$$I_Y(x, y) := I(x, y) - I(x, y - 1)$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} * I$$

$$\text{or } I_X(x, y) := 2I(x, y) - I(x - 1, y) - I(x + 1, y)$$

$$= \begin{pmatrix} -1 & 2 & -1 \end{pmatrix} * I$$

$$I_Y(x, y) := 2I(x, y) - I(x, y - 1) - I(x, y + 1) = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} * I$$

Convolutions / Associativity

- Convolutions are associative:

$$I * (J * K) = (I * J) * K$$

- Example:

First smooth an image with Gaussian w from slide 2,
then compute its x-derivative with $\begin{pmatrix} -1 & 2 & -1 \end{pmatrix}$:

\rightsquigarrow just convolve with $\begin{pmatrix} -1 & 2 & -1 \end{pmatrix} * w$

$$\begin{pmatrix} -1 & 2 & -1 \end{pmatrix} * w = \begin{pmatrix} -0.007 & 0.002 & 0.017 & 0.002 & -0.007 \\ -0.033 & 0.008 & 0.077 & 0.008 & -0.033 \\ -0.054 & 0.077 & 0.128 & 0.077 & -0.054 \\ -0.033 & 0.008 & 0.077 & 0.008 & -0.033 \\ -0.007 & 0.002 & 0.017 & 0.002 & -0.007 \end{pmatrix}$$

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Edges, Corners, and Interest Points

- ▶ good candidates for points that are easy to recognize and match in two images are
 - ▶ points on edges
 - ▶ corners

i.e., points with sudden intensity changes.

- ▶ two stage approach: given an image $I \in \mathbb{R}^{N \times M}$,
 1. compute an interestingness measure $i \in \mathbb{R}^{N \times M}$ for points,
 2. select a useful set of points $p_1, \dots, p_K \in [N] \times [M]$
 - ▶ with high interestingness measure
 - ▶ not too close to each other.
- ▶ many names: corners, interest points, keypoints, salient points, ...

Note: $[N] := \{1, \dots, N\}$.

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Computer Vision 2. Edges, Corners, and Interest Points

Gradient Magnitude (Canny Edge Detector)

- ▶ Simply use the **magnitude of the gradient** as interestingness measure:

$$i(x, y) = \sqrt{(D_X * I)(x, y)^2 + (D_Y * I)(x, y)^2}$$

- ▶ D_X, D_Y : differentiation kernels, e.g.,

$$D_X := \begin{pmatrix} -1 & 2 & -1 \end{pmatrix}, \quad D_Y := \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

Gradient Magnitude / Example

gradient magnitude:

original (grayscale):



overlay with 500 interest points:



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Computer Vision 2. Edges, Corners, and Interest Points

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Laplacian of Gaussian and Difference of Gaussian

Further simple interestingness measures:

- **Laplacian of Gaussian (LoG):**

$$i(x, y) = (((D_X * D_X + D_Y * D_Y) * G) * I)(x, y)$$

- uses second order information

- **Difference of two Gaussians (DoG):**

$$i(x, y) = ((G_{\sigma_1} - G_{\sigma_2}) * I)(x, y), \quad \sigma_1 \neq \sigma_2$$

- uses variations at different scales
- often interpreted as limit of Laplacian of Gaussians

$$((D_X * D_X + D_Y * D_Y) * G_{\sigma}) * I \approx \frac{\sigma}{\Delta\sigma} ((G_{\sigma+\Delta\sigma} - G_{\sigma-\Delta\sigma}) * I)$$

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Harris Corner Detector

- Represent a **corner** by its **patch** surrounding it, represent such a **patch** by a **weight function**

$$w : [N] \times [M] \rightarrow \mathbb{R},$$

i.e.,

$$w(x, y) := \begin{cases} 1, & \text{if } |x - x_0| < 3 \text{ and } |y - y_0| < 3 \\ 0, & \text{else} \end{cases}$$

for a rectangular patch of size 5 centered around (x_0, y_0) .

- A point is easy to identify, if its minimum in the **autocorrelation surface** is pronounced:

$$E(\Delta x, \Delta y; w) := \sum_{x, y} w(x, y) (I(x + \Delta x, y + \Delta y) - I(x, y))^2$$

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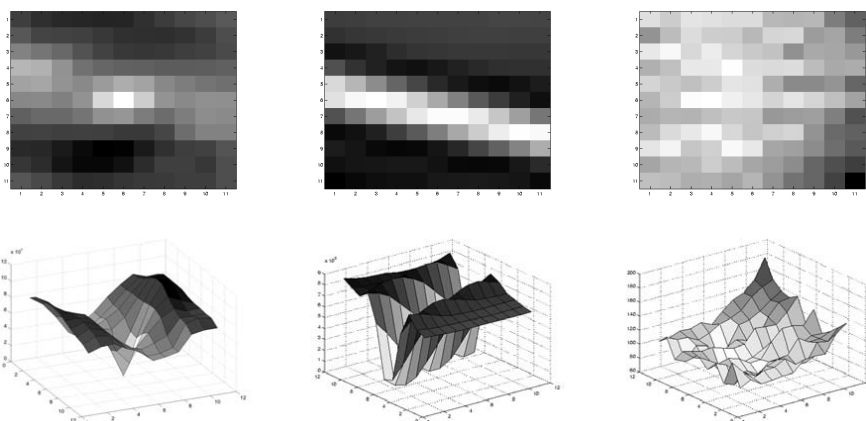
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Computer Vision 2. Edges, Corners, and Interest Points

Harris Corner Detector / Autocorrelation Surface



(a)



Note: left to right: flower bed, roof edge, cloud.

[Sze11, p. 187]

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Harris Corner Detector

$$E(\Delta x, \Delta y; w) := \sum_{x,y} w(x,y) (I(x + \Delta x, y + \Delta y) - I(x,y))^2$$

with Hessian at minimum:

$$H(0,0; w) \approx 2 \sum_{x,y} w(x,y) \nabla I|_{(x,y)} \nabla I|_{(x,y)}^T, \quad \text{for } \frac{\partial^2 I}{\partial^2(x,y)} := 0$$

$$= 2w * \begin{pmatrix} (I_X)^2 & I_X I_Y \\ I_X I_Y & (I_Y)^2 \end{pmatrix},$$

$$I_X(x,y) := I(x+1,y) - I(x,y) \approx \frac{\partial I}{\partial x}(x,y)$$

$$I_Y(x,y) := I(x,y+1) - I(x,y) \approx \frac{\partial I}{\partial y}(x,y)$$

Note: $I * J(x,y) := \sum_{x',y'} I(x-x', y-y') J(x', y')$ **convolution** of two images.

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Harris Corner Detector

use SVD to assess steepness

$$H = U \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} U^T, \quad \sigma_1 \geq \sigma_2 \geq 0, \quad UU^T = I$$

and define interestingness measure:

$$i_{\text{Shi-Tomasi}}(x,y) := \sigma_2$$

$$i_{\text{Harris}}(x,y) := \sigma_1 \sigma_2 - \alpha (\sigma_1 + \sigma_2)^2 = \det H - \alpha \text{trace}(H)^2, \quad \alpha := 0.06$$

$$i_{\text{Triggs}}(x,y) := \sigma_2 - \alpha \sigma_1, \quad \alpha := 0.05$$

$$i_{\text{Brown}}(x,y) := \sigma_1 \sigma_2 / (\sigma_1 + \sigma_2) = \det H / \text{trace}(H)$$

- the larger $\sigma_{1:2}$, the steeper the autocorrelation surface E .
- Harris and Brown avoid computing σ_1, σ_2 explicitly (which requires computing a square root).

Harris Corner Detector / Algorithm

```

1: procedure INTERESTPOINTS-HARRIS( $I \in \mathbb{R}^{N \times M}$ ;  $w \in \mathbb{R}^{-K:K \times -L:L}$ ,  $\alpha \in \mathbb{R}$ )
2:    $I_X := D_X * I$ 
3:    $I_Y := D_Y * I$ 
4:    $I_X^2 := I_X \cdot I_X$ 
5:    $I_Y^2 := I_Y \cdot I_Y$ 
6:    $I_X I_Y := I_X \cdot I_Y$ 
7:    $A := w * I_X^2$ 
8:    $B := w * I_Y^2$ 
9:    $C := w * I_X I_Y$ 
10:   $i := A \cdot B - C \cdot C - \alpha(A + B) \cdot (A + B)$ 
11:  return  $i$ 

```

▷ compute $H(x, y) = \begin{pmatrix} A(x, y) & C(x, y) \\ C(x, y) & B(x, y) \end{pmatrix}$

► D_X, D_Y : differentiation kernels, e.g.,

$$D_X := \begin{pmatrix} -1 & 2 & -1 \end{pmatrix}, D_Y := \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}.$$

Note: \cdot denotes the element/pixelwise product.

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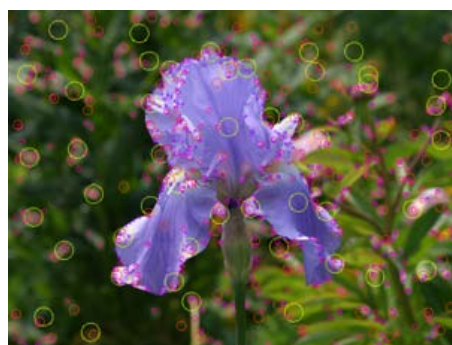
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Computer Vision 2. Edges, Corners, and Interest Points

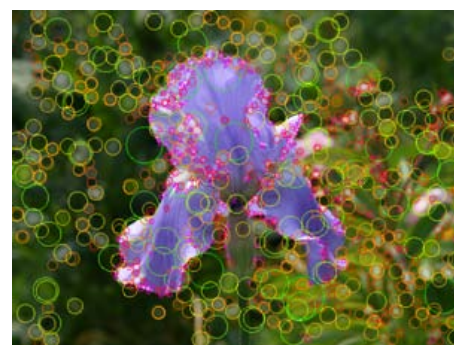
Harris Corner Detector / Example



(a)



(b)



(c)

a) original, b) Harris corners, c) DoG interest points

[Sze11, p. 213]

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Interest Points at Different Scales (SIFT Detector)

- Interest points also can be identified **at different scales** in parallel:

$$i(p, s) := (G_{\sigma_{s+1}} * I - G_{\sigma_s} * I), \quad s \in [S]$$

where

$$\sigma_1 > \sigma_2 > \dots > \sigma_S$$

where $S \in \mathbb{N}$ is the **number of scale levels**

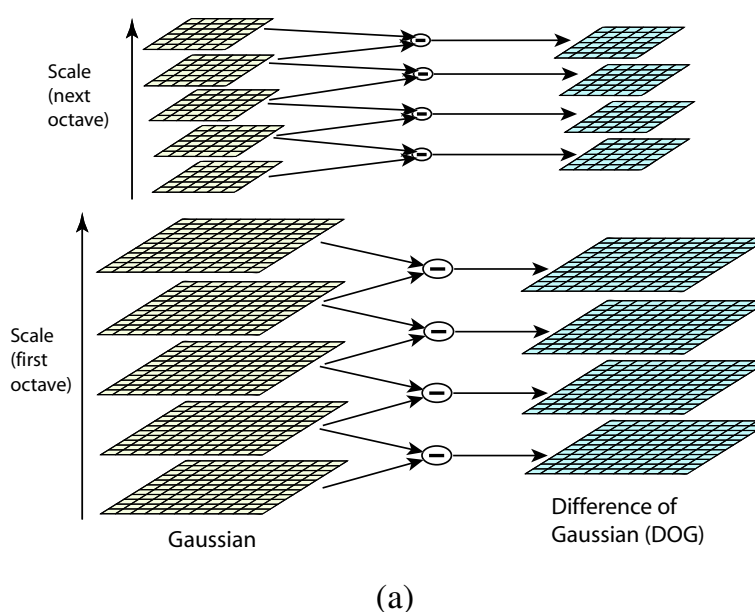
- Often scale levels are grouped by octaves:
 - each octave is represented by a downsampling by a factor 2
 - scales within an octave are $\sigma_s := 2^{s/S_o} \sigma$
(with S_o the number of scale levels within an octave)

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Computer Vision 2. Edges, Corners, and Interest Points

Interest Points at Different Scales (SIFT Detector)



[Sze11, p. 216]

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Non-Maximum Suppression

- Often neighbors of interest points have similar high interestingness, yielding redundant close-by interest points.
- Keep only interest points that are **local maxima** in their neighborhood:

$$i'(p) := \begin{cases} i(p), & \text{if } i(p) > i(p') \forall p' \in N(p) \\ 0, & \text{else} \end{cases}, \quad p \in [N] \times [M]$$

with **neighborhood**

$$N_K(p) := \{p' \in [N] \times [M] \mid |p_x - p'_x| \leq K, |p_y - p'_y| \leq K, p' \neq p\} \quad \text{recta}$$

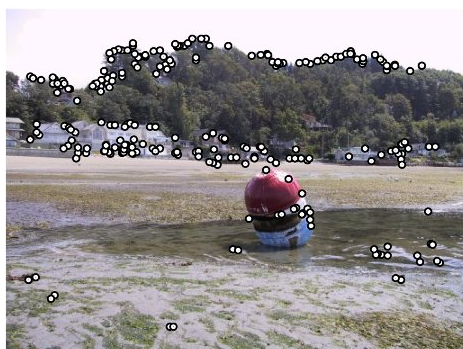
$$N_K(p) := \{p' \in [N] \times [M] \mid \|p - p'\| \leq K, p' \neq p\} \quad \text{circu}$$

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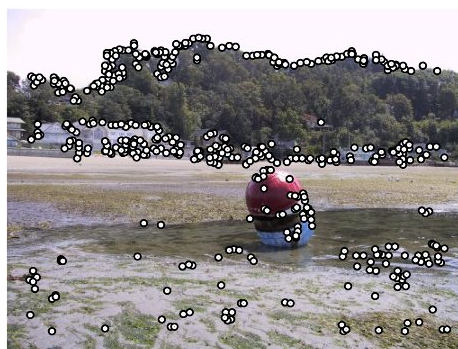
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Computer Vision 2. Edges, Corners, and Interest Points

Non-Maximum Suppression / Example



(a) Strongest 250



(b) Strongest 500



(c) ANMS 250, $r = 24$



(d) ANMS 500, $r = 16$

Note: ANMS = adaptive non-maximum suppression; see the book for details. [Sze11, p. 214]

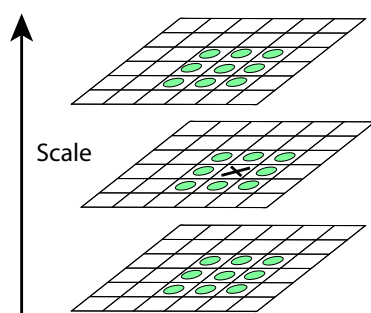
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Non-Maximum Suppression / At Different Scale

- Non-Maximum Suppression also can be extended to work on interest points at different scale:

$$N_K(p, s) := \{(p', s') \in [N] \times [M] \times [S] \mid |p_x - p'_x| \leq K, |p_y - p'_y| \leq K, |s - s'| \leq 1, p' \neq p\}$$



[Sze11, p. 216]

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Computer Vision 2. Edges, Corners, and Interest Points

SIFT Interest Points

SIFT refines interest points by further steps:

- non-maximum suppression at different scale
- localization of interest points at sub-pixel granularity

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Computer Vision 3. Image Patch Descriptors

Image Patch Descriptors

- ▶ Which properties from a patch to extract?
 - ▶ grayscale intensities, color intensities, gradient directions
- ▶ Which patches to extract?
 - ▶ orientation of the patch w.r.t. the image frame
 - ▶ offset of the patch w.r.t. the interest point (cells)

Histograms

- ▶ the most simple patch:
 - ▶ a square centered on the interest point
- ▶ properties:
 - ▶ most simple: grayscale **intensities** of the pixels
 - ▶ is affected by global intensity fluctuations
 - ▶ **gradient directions**
- ▶ how to represent?
 - ▶ as a matrix or a vector
 - ▶ is affected by rotations
 - ▶ by some scalar properties (mean, standard deviation)
 - ▶ represents only little information
 - ▶ by its **histogram**

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Computer Vision 3. Image Patch Descriptors

Histograms / Intensities

- ▶ represent interest point (x, y) by its B -dimensional **intensity histogram features** $\phi(x, y)$:

$$\phi(x, y)_b := |\{(x', y') \in \mathcal{N}(x, y) \mid I(x', y') \in \text{bin}_b\}|, \quad b = 0, \dots, B - 1$$

$$\text{bin}_b := \left[\frac{b}{B} I_{\max}, \frac{b+1}{B} I_{\max} \right$$

$$\mathcal{N}(x, y) := \{(x', y') \in [N] \times [M] \mid |x' - x| < K, |y' - y| < K\}$$

for intensities $I(x, y)$ in range $[0, I_{\max}]$.

Histograms / Smoothed Counting

- ▶ To avoid non-continuous changes if a value crosses bin boundaries, values can be counted
 - ▶ in both closest bins,
 - ▶ antiproportional to their distance from the bin center

$$\text{binc}_b := \frac{b + 0.5}{B} I_{\max}$$

$$\text{bin}_b := \sum_{(x', y') \in \mathcal{N}(x, y)} \max\left(0, 1 - \frac{|I(x', y') - \text{binc}_b|}{I_{\max}/B}\right)$$

- ▶ sometimes called **trilinear counting**.

Histograms / Gradient Directions

- ▶ represent interest point (x, y) by its B -dimensional **gradient directions histogram features** $\phi(x, y)$:

$$\phi(x, y)_b := |\{(x', y') \in \mathcal{N}(x, y) \mid d(x', y') \in \text{bin}_b\}|, \quad b = 0, \dots, B - 1$$

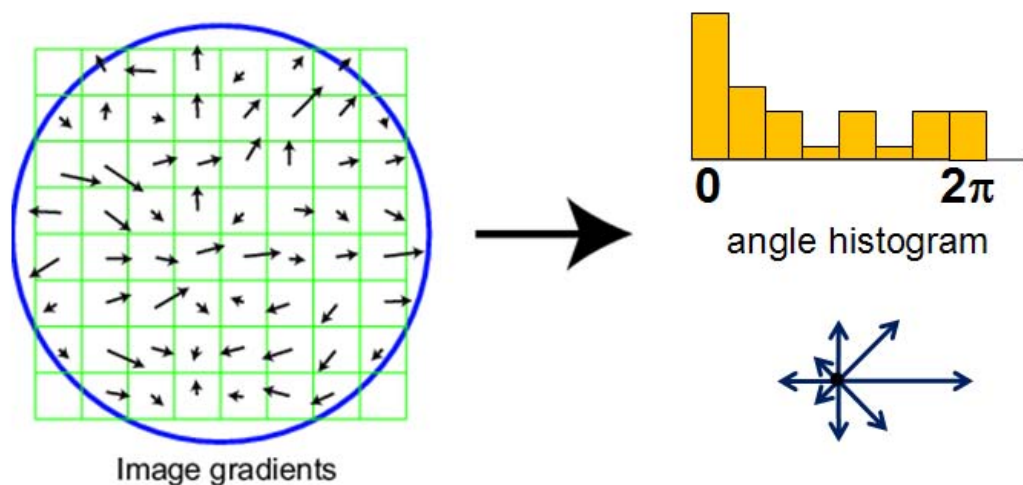
$$d(x, y) := \tan^{-1}((D_Y * I)(x, y) / (D_X * I)(x, y))$$

$$\text{bin}_b := \left[\frac{b}{B} 2\pi, \frac{b+1}{B} 2\pi \right[$$

- ▶ variant: weight gradients by their magnitude:

$$\phi(x, y)_b := \sum_{(x', y') \in \mathcal{N}(x, y), d(x', y') \in \text{bin}_b} ((D_X * I)(x', y'))^2 + ((D_Y * I)(x', y'))^2$$

Histograms / Gradients / Example



[Sze11, p. 217]

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Block Descriptors

- Describe an interest point not just by features of the surrounding patch,
but by the features of several neighboring patches (**blocks, cells**):

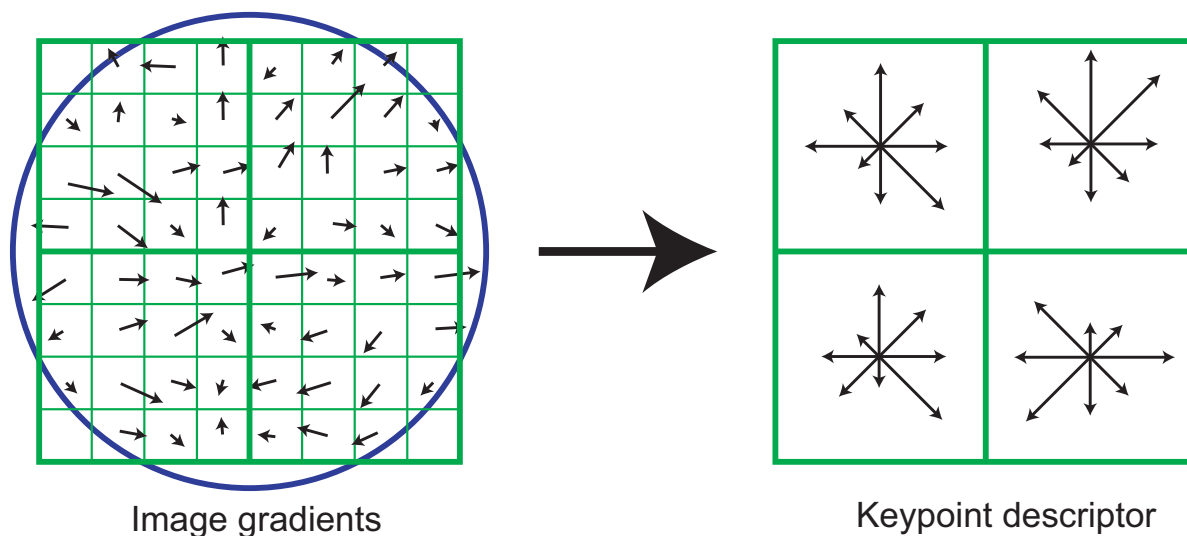
$$\phi(x, y) := \bigoplus_{(x', y') \in \mathcal{C}(x, y)} \phi'(x', y')$$

$$\mathcal{C}(x, y) := \{x + c\Delta X, y + d\Delta Y \mid c, d \in \{-C, \dots, C\}\}$$

- Often a simple partition of a large $(2C + 1)(2K + 1) \times (2C + 1)(2K + 1)$ patch is used ($\Delta X = \Delta Y = 2K + 1$).
- Features have dimensions $(2C + 1)^2 B$.

Note: $(x_1, \dots, x_N) \oplus (y_1, \dots, y_M) := (x_1, \dots, x_N, y_1, \dots, y_M)$ **concatenation**.

Block Descriptors



[Low04, p. 15]

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Computer Vision 3. Image Patch Descriptors

Align Patches by the Gradient Direction of the Interest Point

- Extract features from the image rotated by
 - the negative gradient direction at the interest point
 - around the interest point

(afterwards the gradient at the interest point (x, y) points towards positive x -direction):

$$\psi := -d(x, y)$$

$$R_{\psi}(x', y') := \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix} \left(\begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

$$I_{bi}(x, y) := (1 - (x - \lfloor x \rfloor))(1 - (y - \lfloor y \rfloor)) I(\lfloor x \rfloor, \lfloor y \rfloor) \\
+ (x - \lfloor x \rfloor)(1 - (y - \lfloor y \rfloor)) I(\lceil x \rceil, \lfloor y \rfloor) \\
+ (1 - (x - \lfloor x \rfloor))(y - \lfloor y \rfloor) I(\lfloor x \rfloor, \lceil y \rceil) \\
+ (x - \lfloor x \rfloor)(y - \lfloor y \rfloor) I(\lceil x \rceil, \lceil y \rceil)$$

(bilinear interpolation)

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SIFT descriptors

- ▶ patches:
 - ▶ extract from the scaled image the interest point has been detected on
 - ▶ align patch by the gradient direction of the interest point
 - ▶ 16×16 , partitioned into 16 blocks a 4×4
- ▶ block features:
 - ▶ gradient directions
 - ▶ weighted by a Gaussian of the distance to the interest point
- ▶ block feature aggregation:
 - ▶ smoothly counted histograms
 - ▶ 8 bins
- ▶ \rightsquigarrow feature vector $\phi \in \mathbb{R}^{128}$
- ▶ normalization in 3 steps:

$$\phi'_i := \phi_i / \|\phi\|_2, \quad \phi''_i := \min(0.2, \phi'_i), \quad \phi'''_i := \phi''_i / \|\phi''\|_2$$

Image Descriptors

To describe a whole image (not just a patch),
two main approaches are used:

1. Concatenate patch descriptors of equally spaced “interest points”
 - 1.1 e.g., used in **Histograms of Oriented Gradients (HoG)**
2. **Bag of words descriptors:**
 - 2.1 compute interest points and their descriptors for a set of images
 - 2.2 discretize the descriptors
 - ▶ e.g., clustering in K clusters using k-means
 - 2.3 represent each image by the K cluster frequencies of their interest point descriptors

Histograms of Oriented Gradients (HoG)

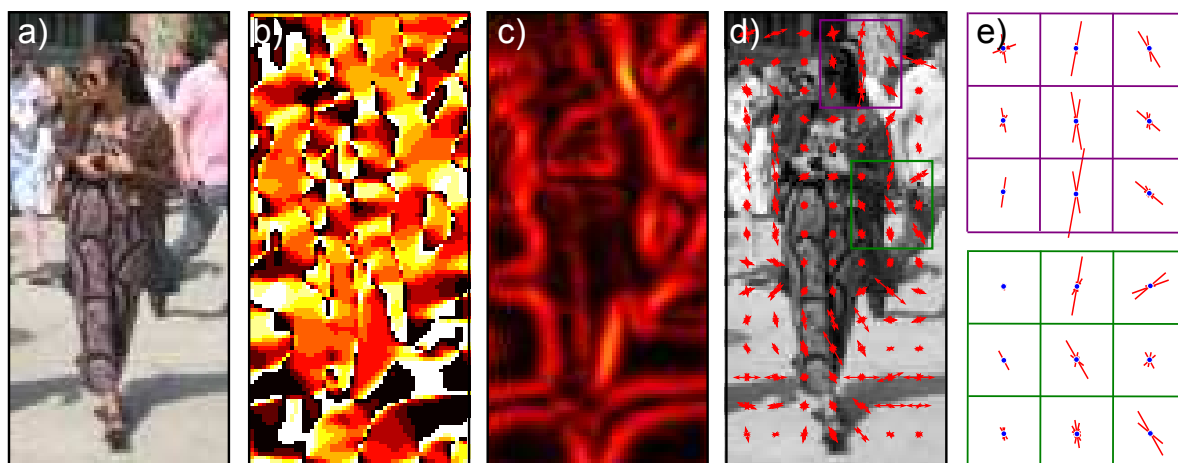


Figure 13.17 HOG descriptor. a) Original image. b) Gradient orientation, quantized into nine bins from 0 to 180° . c) Gradient magnitude. d) Cell descriptors are 9D orientation histograms that are computed within 6×6 pixel regions. e) Block descriptors are computed by concatenating 3×3 blocks of cell descriptors. The block descriptors are normalized. The final HOG descriptor consists of the concatenated block descriptors.

[Pri12, p. 343]

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Computer Vision 4. Interest Point Matching

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Settings, Assumptions, Distances

Two settings:

- ▶ match interest points in different scenes
 - ▶ goal: detect similar objects (object identification)
 - ▶ coordinates of the points do not matter

$$d\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) := d'(\phi(x_1, y_1), \phi(x_2, y_2)) = \|\phi(x_1, y_1) - \phi(x_2, y_2)\|_2$$

- ▶ match interest points in two views of the same scene
 - ▶ goal: detect corresponding points in different views of the same scene (required for SLAM)
 - ▶ coordinates of corresponding points also should be close, e.g.,

$$\begin{aligned} d\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) &:= \alpha d'\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) + \beta d'(\phi(x_1, y_1), \phi(x_2, y_2)) \\ &= \alpha \left\| \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right\|_2 + \beta \|\phi(x_1, y_1) - \phi(x_2, y_2)\|_2 \end{aligned}$$

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Computer Vision 4. Interest Point Matching

Simple methods

To match two sets P and Q of interest points:

- ▶ match interest points by **distance threshold**

$$p \sim q :\Leftrightarrow d(p, q) < d_{\max}, \quad p \in P, q \in Q$$

- ▶ distance threshold d_{\max} can be estimated from known matches and non-matches

- ▶ match interest points by **nearest neighbor**

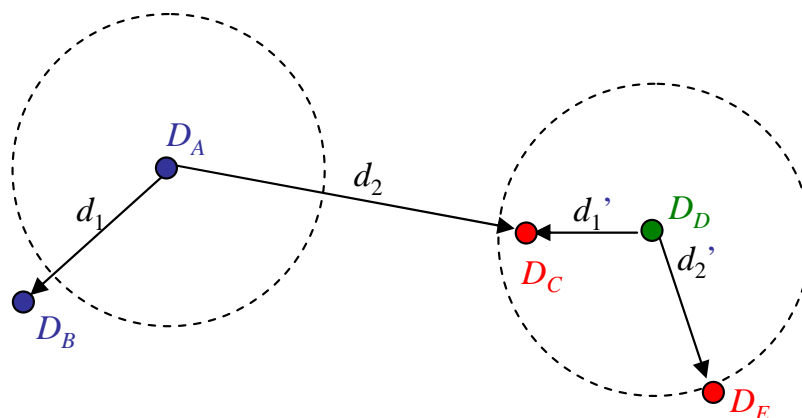
$$p \sim q :\Leftrightarrow q = \arg \min_{q \in Q} d(p, q)$$

Nearest Neighbor Distance Ratio

- match interest points by **nearest neighbor distance ratio** (NNDR)

$p \sim q : \Leftrightarrow i) q = \arg \min_{q \in Q} d(p, q)$ and

ii) $\text{NNDR}(p, q) := \frac{d(p, q)}{d(p, q')} < \text{NNDR}_{\min}, \quad q' := \arg \min_{q' \in Q \setminus \{q\}} d(p, q')$



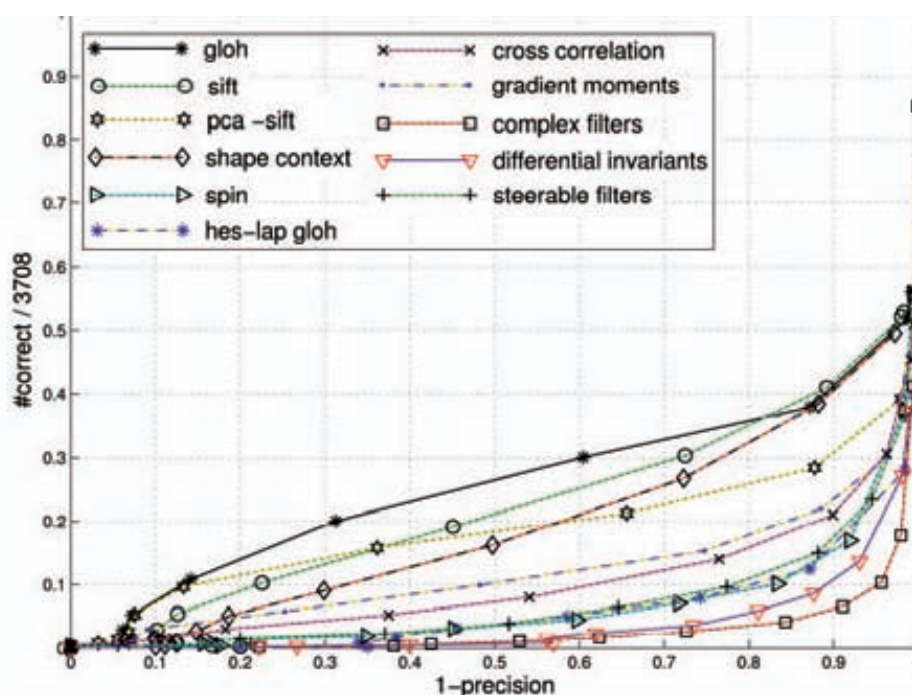
[Sze11, p. 228]

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Comparison of Different Descriptors & Matchings

a) fixed threshold:



[Sze11, p. 229]

b) nearest neighbor:

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Mutual Nearest Neighbors

- match interest points if they **mutually** are nearest neighbors

$$p \sim q : \Leftrightarrow i) \ q = \arg \min_{q \in Q} d(p, q) \text{ and}$$

$$ii) \ p = \arg \min_{p \in P} d(p, q)$$

- also for more than two views P_1, P_2, \dots, P_V
(called **closed chains**)

(p_1, p_2, \dots, p_V) corresponding tuple

$$: \Leftrightarrow i) \ p_{v+1} = \arg \min_{q \in P_{v+1}} d(p_v, q), \quad v = 1, \dots, V-1 \text{ and}$$

$$ii) \ p_1 = \arg \min_{q \in P_V} d(p_V, q)$$

Outline

1. Smoothing, Image Derivatives, Convolutions
2. Edges, Corners, and Interest Points
3. Image Patch Descriptors
4. Interest Point Matching
5. A Simple Application: Image Stitching

Image Stitching

- ▶ join several images depicting overlapping parts of the same real scene to one large image
- ▶ algorithm:
 1. detect interest points in all images and extract their descriptors
 2. match interest points between every two images
 3. form a tree linking the best matching image pairs
 4. estimate a similarity transform between each two such images
 5. transform all images to joint coordinates
 6. average overlapping image regions
- ▶ also called **panography**

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Computer Vision 5. A Simple Application: Image Stitching

Image Stitching / Example



[Sze11, p. 312]

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Image Stitching / Different Transforms



(a) translation [2 dof]



(b) affine [6 dof]



(c) perspective [8 dof]



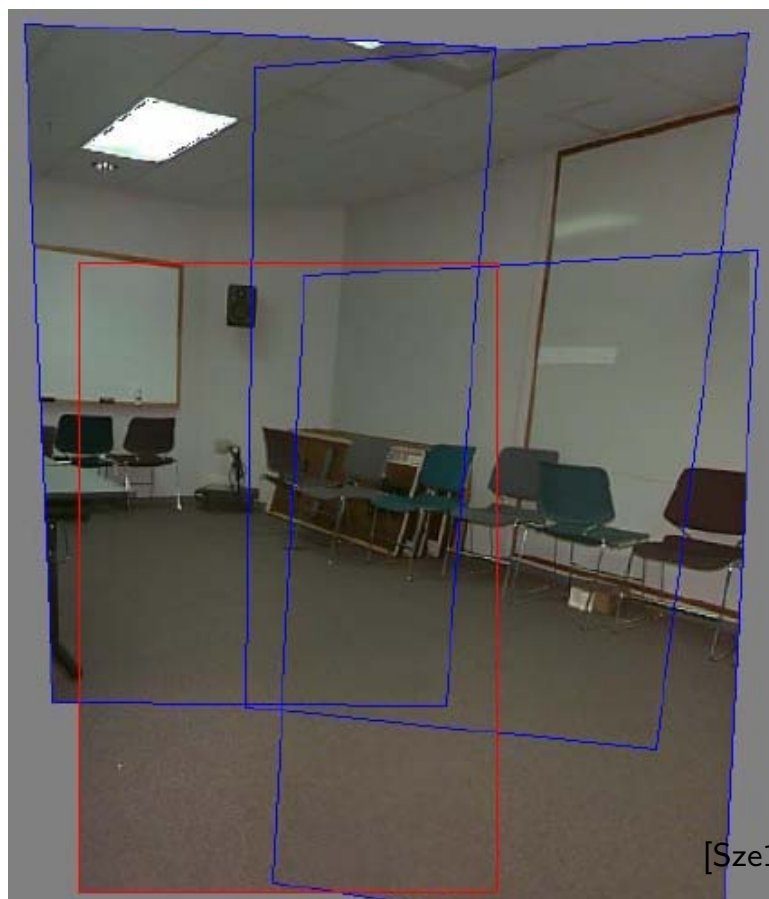
(d) 3D rotation [3+ d

[Sze11, p. 425]

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Image Stitching / Example



[Sze11, p. 330]

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Summary

- ▶ Small intensity fluctuations can be damped by **smoothing**, intensity changes can be captured by **image derivatives**, both being **convolutions**.
- ▶ Interest points are found as maxima of an **interestingness measure**,
 - ▶ **gradient magnitude**, **Laplacian of Gaussian (LoG)**, **Different of two Gaussians (DoG)**
 - ▶ **Harris corners**:
 - ▶ large eigenvalues of the Hessian
 - ▶ can be approximated efficiently: $\det H - \alpha(\text{trace} H)^2$
 - ▶ **SIFT**:
 - ▶ detected interest points at different scale
 - ▶ several further tweaks
- ▶ **non-maximum suppressions**:
ignore large values in the vicinity of a maximum

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Computer Vision 5. A Simple Application: Image Stitching

Summary (2/3)

- ▶ Interest points are characterized by local image information (**descriptors**)
- ▶ Descriptors often describe several patches (**blocks/cells**)
- ▶ Patches are described by **histograms**
- ▶ Histograms usually do not count pixel intensities, but **gradient directions**
- ▶ Descriptors sometimes
 - ▶ align patches with the orientation of the gradient at the interest point
 - ▶ weight gradient directions by their
 - ▶ gradient magnitude and/or
 - ▶ distance of the location to the interest point
- ▶ Common descriptors:
 - ▶ **SIFT descriptors**, **Histogram of Gradients (HoG)**

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Summary (3/3)

- ▶ Whole images can be described two ways:
 - ▶ by the descriptors on a fixed grid of “interest points”
 - ▶ by the cluster frequencies of descriptors of variably located interest points

Both is useful, e.g. for image classification.

- ▶ Interest points are **matched** by their descriptors
 - ▶ for geometric tasks: also by their positions
- ▶ To match interest points, **nearest neighbors** are used
 - ▶ with a maximal distance threshold to avoid wrong matches e.g. of points occluded in one view
 - ▶ **Nearest Neighbor Distance Ratio**
 - ▶ **mutual nearest neighbors, closed chains** in multiple views.
- ▶ **Corresponding points** can be used for
 - ▶ **image stitching**
 - ▶ SLAM, camera auto-calibration, ...

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Computer Vision

Further Readings

- ▶ Interest points and patch descriptors: [Pri12, ch. 13], [Sze11, ch. 4].
- ▶ Image stitching: [Sze11, ch. 9].

References



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