Computer Vision

4. Interest Points

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Outline

1. Smoothing, Image Derivatives, Convolutions
2. Edges, Corners, and Interest Points
3. Image Patch Descriptors
4. Interest Point Matching
5. A Simple Application: Image Stitching

Outline

Smoothing / Blurring / Averaging

- **Smoothing**: Replace each pixel by the weighted average of its surrounding patch:

$$I_{\text{smooth}}(x, y; w) := \sum_{\Delta x, \Delta y} w(-\Delta x, -\Delta y) I(x + \Delta x, y + \Delta y)$$
$$= \sum_{x', y'} w(x - x', y - y') I(x', y')$$

  - **padding** with 0 at the image boundaries.
- example: **box kernel**

$$w_{-2:2,-2:2}(\Delta x, \Delta y) := \frac{1}{25} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

- **Gaussian smoothing**: smoothing with a Gaussian kernel.

Gaussian Kernels

- Precomputed weights: (clipped) Gaussian density values

$$\tilde{w}(\Delta x, \Delta y) := \begin{cases} \mathcal{N}(\sqrt{\Delta x^2 + \Delta y^2}; 0, \sigma^2), & \text{if } |\Delta x| \leq K, |\Delta y| \leq K \\ 0, & \text{else} \end{cases}$$

$$w(\Delta x, \Delta y) := \frac{\tilde{w}(\Delta x, \Delta y)}{\sum_{\Delta x', \Delta y'} \tilde{w}(\Delta x, \Delta y)}$$

- clipped: small support, **window size K**.
- example ($K = 2, \sigma^2 = 1$):

$$w_{-2:2,-2:2} := \begin{pmatrix} 0.003 & 0.013 & 0.022 & 0.013 & 0.003 \\ 0.013 & 0.060 & 0.098 & 0.060 & 0.013 \\ 0.022 & 0.098 & 0.162 & 0.098 & 0.022 \\ 0.013 & 0.060 & 0.098 & 0.060 & 0.013 \\ 0.003 & 0.013 & 0.022 & 0.013 & 0.003 \end{pmatrix}$$

Note: $\mathcal{N}(x; \mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

Image Derivatives

- **Image Derivative**: How does the intensity values change in x or y direction?

$$I_X(x,y) := I(x,y) - I(x-1,y)$$
$$I_Y(x,y) := I(x,y) - I(x,y-1)$$

or symmetric

$$I_X(x,y) := 2I(x,y) - I(x-1,y) - I(x+1,y)$$
$$I_Y(x,y) := 2I(x,y) - I(x,y-1) - (x,y-2)$$

Convolutions

- Smoothing, Image Derivatives and further operations such as filtering can be represented by a
  - **convolution**: an image where each pixel (x, y) represents the weighted sum around (x, y) in image I weighted with w:

$$(w * I)(x, y) := \sum_{x', y'} w(x - x', y - y') I(x', y')$$

- Examples:

$$\begin{aligned}
I_{\text{smooth}} & & &= w * I \\
I_X(x, y) &:= I(x, y) - I(x - 1, y) & &= \begin{pmatrix} 1 & -1 \end{pmatrix} * I \\
I_Y(x, y) &:= I(x, y) - I(x, y - 1) & &= \begin{pmatrix} 1 \\ -1 \end{pmatrix} * I \\
\text{or } I_X(x, y) &:= 2I(x, y) - I(x - 1, y) - I(x + 1, y) & &= \begin{pmatrix} -1 & 2 & -1 \end{pmatrix} * \\
I_Y(x, y) &:= 2I(x, y) - I(x, y - 1) - (x, y - 2) & &= \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} * I
\end{aligned}$$

Convolutions / Associativity

- Convolutions are associative:

$$I * (J * K) = (I * J) * K$$

- Example:
  First smooth an image with Gaussian w from slide 2,
  then compute its x-derivative with $\begin{pmatrix} -1 & 2 & -1 \end{pmatrix}$:
  $\rightsquigarrow$ just convolve with $\begin{pmatrix} -1 & 2 & -1 \end{pmatrix} * w$

$$\begin{pmatrix} -1 & 2 & -1 \end{pmatrix} * w = \begin{pmatrix} -0.007 & 0.002 & 0.017 & 0.002 & -0.007 \\ -0.033 & 0.008 & 0.077 & 0.008 & -0.033 \\ -0.054 & 0.077 & 0.128 & 0.077 & -0.054 \\ -0.033 & 0.008 & 0.077 & 0.008 & -0.033 \\ -0.007 & 0.002 & 0.017 & 0.002 & -0.007 \end{pmatrix}$$

Outline

Edges, Corners, and Interest Points

- good candidates for points that are easy to recognize and match in two images are
  - points on edges
  - corners

  i.e., points with sudden intensity changes.

- two stage approach: given an image $I \in \mathbb{R}^{N \times M}$,
  1. compute an interestingness measure $i \in \mathbb{R}^{N \times M}$ for points,
  2. select a useful set of points $p_1, \ldots, p_K \in [N] \times [M]$
     - with high interestingness measure
     - not too close to each other.

- many names: corners, interest points, keypoints, salient points, ...

Note: $[N] := \{1, \ldots, N\}$.

Gradient Magnitude (Canny Edge Detector)

- Simply use the **magnitude of the gradient** as interestingness measure:

$$i(x,y) = \sqrt{(D_X * I)(x,y)^2 + (D_Y * I)(x,y)^2}$$

- D_X, D_Y: differentiation kernels, e.g.,

$$D_X := \begin{pmatrix} -1 & 2 & -1 \end{pmatrix}, \quad D_Y := \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

Laplacian of Gaussian and Difference of Gaussian

Further simple interestingness measures:

- **Laplacian of Gaussian** (**LoG**):

$$i(x,y) = (((D_X * D_X + D_Y * D_Y) * G) * I)(x,y)$$

  - uses second order information

- **Difference of two Gaussians** (**DoG**):

$$i(x,y) = ((G_{\sigma_1} - G_{\sigma_2}) * I)(x,y), \quad \sigma_1 \neq \sigma_2$$

  - uses variations at different scales
  - often interpreted as limit of Laplacian of Gaussians

$$((D_X * D_X + D_Y * D_Y) * G_\sigma) * I \approx \frac{\sigma}{\Delta\sigma}((G_{\sigma+\Delta\sigma} - G_{\sigma-\Delta\sigma}) * I)$$

Harris Corner Detector

- Represent a **corner** by its **patch** surrounding it, represent such a **patch** by a **weight function**

$$w : [N] \times [M] \to \mathbb{R},$$

i.e.,

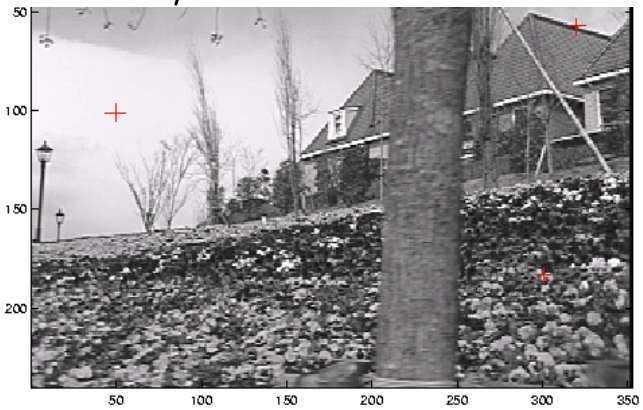
$$w(x, y) := \begin{cases} 1, & \text{if } |x - x_0| < 3 \text{ and } |y - y_0| < 3 \\ 0, & \text{else} \end{cases}$$

for a rectangular patch of size 5 centered around (x_0, y_0).

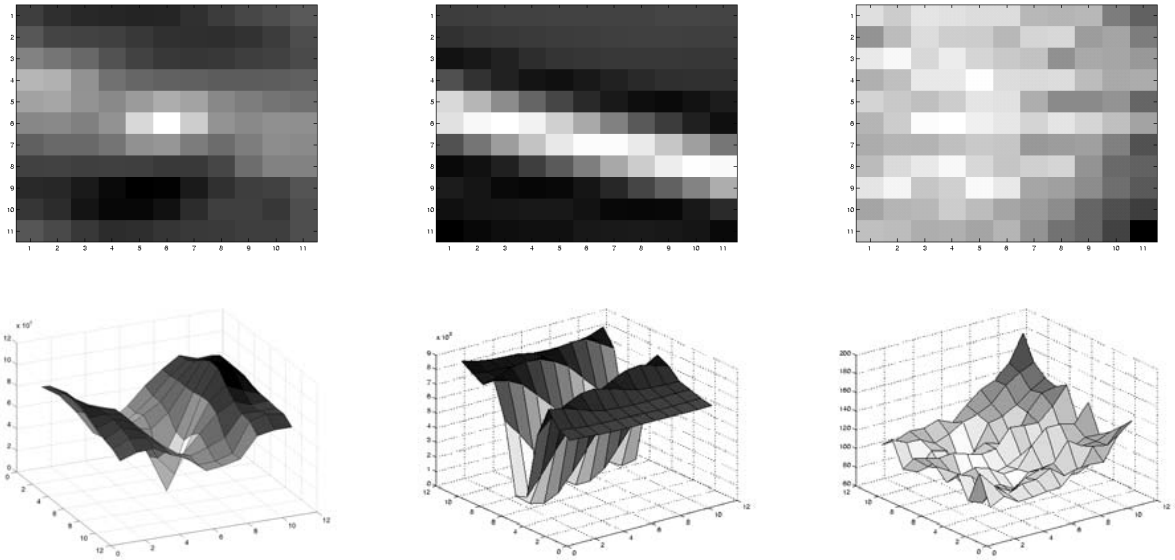
- A point is easy to identify, if its minimum in the **autocorrelation surface** is pronounced:

$$E(\Delta x, \Delta y; w) := \sum_{x,y} w(x, y)(I(x + \Delta x, y + \Delta y) - I(x, y))^2$$

Harris Corner Detector / Autocorrelation Surface



(a)



Note: left to right: flower bed, roof edge, cloud.

[Sze11, p. 187]

Harris Corner Detector

$$E(\Delta x, \Delta y; w) := \sum_{x,y} w(x,y)(I(x+\Delta x, y+\Delta y) - I(x,y))^2$$

with Hessian at minimum:

$$H(0,0;w) \approx 2 \sum_{x,y} w(x,y) \nabla I|_{(x,y)} \nabla I|_{(x,y)}^T, \quad \text{for } \frac{\partial^2 I}{\partial^2 (x,y)} := 0$$

$$= 2w * \begin{pmatrix} (I_X)^2 & I_X I_Y \\ I_X I_Y & (I_Y)^2 \end{pmatrix},$$

$$I_X(x,y) := I(x+1,y) - I(x,y) \approx \frac{\partial I}{\partial x}(x,y)$$

$$I_Y(x,y) := I(x,y+1) - I(x,y) \approx \frac{\partial I}{\partial y}(x,y)$$

Note: $I * J(x,y) := \sum_{x',y'} I(x-x', y-y') J(x',y')$ **convolution** of two images.

Harris Corner Detector

use SVD to assess steepness

$$H = U \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} U^T, \quad \sigma_1 \geq \sigma_2 \geq 0, UU^T = I$$

and define interestingness measure:

$$i_{\text{Shi-Tomasi}}(x, y) := \sigma_2$$

$$i_{\text{Harris}}(x, y) := \sigma_1\sigma_2 - \alpha(\sigma_1 + \sigma_2)^2 = \det H - \alpha\,\text{trace}(H)^2, \qquad \alpha := 0.06$$

$$i_{\text{Triggs}}(x, y) := \sigma_2 - \alpha\sigma_1, \qquad \alpha := 0.05$$

$$i_{\text{Brown}}(x, y) := \sigma_1\sigma_2/(\sigma_1 + \sigma_2) = \det H/\text{trace}(H)$$

- the larger $\sigma_{1:2}$, the steeper the autocorrelation surface E.
- Harris and Brown avoid computing σ_1, σ_2 explicitly (which requires computing a square root).

Harris Corner Detector / Algorithm

procedure INTERESTPOINTS-HARRIS($I \in \mathbb{R}^{N \times M}; w \in \mathbb{R}^{-K:K \times -L:L}, \alpha \in \mathbb{R}$)

$I_X := D_X * I$

$I_Y := D_Y * I$

$I_X^2 := I_X \cdot I_X$

$I_Y^2 := I_Y \cdot I_Y$

$I_X I_Y := I_X \cdot I_Y$

$A := w * I_X^2$ ▷ compute $H(x,y) = \begin{pmatrix} A(x,y) & C(x,y) \\ C(x,y) & B(x,y) \end{pmatrix}$

$B := w * I_Y^2$

$C := w * I_X I_Y$

$i := A \cdot B - C \cdot C - \alpha(A+B) \cdot (A+B)$

return i

- D_X, D_Y: differentiation kernels, e.g.,

$$D_X := \begin{pmatrix} -1 & 2 & -1 \end{pmatrix}, D_Y := \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}.$$

Note: $\cdot$ denotes the element/pixelwise product.

Interest Points at Different Scales (SIFT Detector)

- Interest points also can be identified **at different scales** in parallel:

$$i(p,s) := (G_{\sigma_{s+1}} * I - G_{\sigma_s} * I), \quad s \in [S]$$

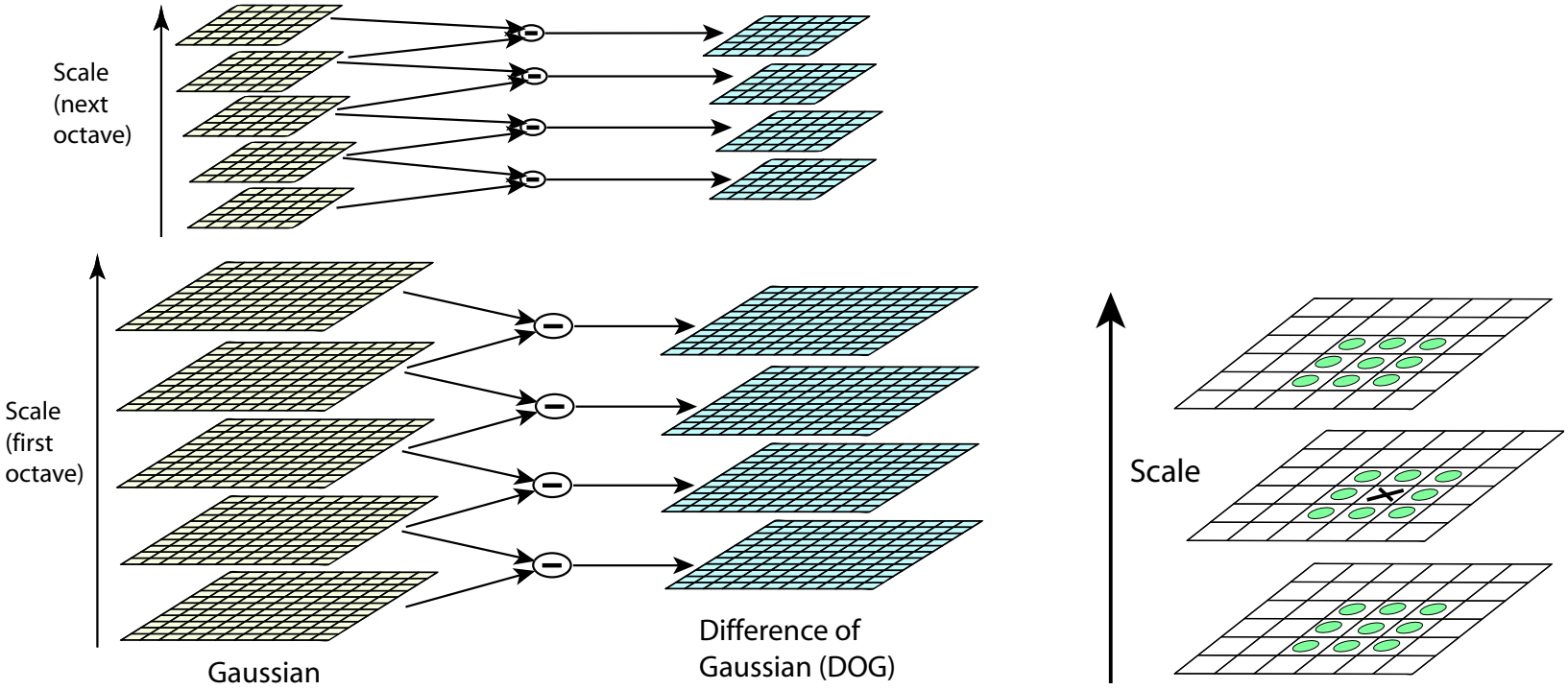
where

$$\sigma_1 > \sigma_2 > \cdots > \sigma_S$$

where $S \in \mathbb{N}$ is the **number of scale levels**

- Often scale levels are grouped by octaves:
  - each octave is represented by a downsampling by a factor 2
  - scales within an octave are $\sigma_s := 2^{s/S_o}\sigma$
    (with S_o the number of scale levels within an ocatve)

Interest Points at Different Scales (SIFT Detector)



(a) (b)

[Sze11, p. 216]

Non-Maximum Suppression

- Often neighbors of interest points have similar high interestingness, yielding redundant close-by interest points.

- Keep only interest points that are **local maxima** in their neighborhood:

$$i'(p) := \begin{cases} i(p), & \text{if } i(p) > i(p') \; \forall p' \in N(p) \\ 0, & \text{else} \end{cases}, \quad p \in [N] \times [M]$$

with **neighborhood**

$$N_K(p) := \{p' \in [N] \times [M] \mid |p_x - p'_x| \leq K, |p_y - p'_y| \leq K, p' \neq p\} \quad \text{recta}$$

$$N_K(p) := \{p' \in [N] \times [M] \mid ||p - p'|| \leq K, p' \neq p\} \quad \text{circu}$$

Non-Maximum Suppression / At Different Scale

- Non-Maximum Suppression also can be extended to work on interest points at different scale:

$$N_K(p,s) := \{(p',s') \in [N] \times [M] \times [S] \mid |p_x - p'_x| \leq K, |p_y - p'_y| \leq K, \\ |s - s'| \leq 1, p' \neq p\}$$

Non-Maximum Suppression / At Different Scale

- SIFT refines interest points by further steps:
  - localization at sub-pixel granularity

Outline

Histograms

SIFT descriptors

Histograms of Oriented Gradients (HoG)

Bag of Words Descriptors

?? Dimensionality Reduction

Outline

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Summary

- ...

Further Readings

- Interest points and patch descriptors: [Pri12, ch. 13], [Sze11, ch. 4].
- Image stitching: [Sze11, ch. 9].

References

Simon JD Prince.
Computer vision: models, learning, and inference.
Cambridge University Press, 2012.

Richard Szeliski.
Computer vision: algorithms and applications.
Springer Science & Business Media, 2011.