Computer Vision
Exercise Sheet 3

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Exercise 1: Homogeneous Coordinates (6 points)

a) Three planes through the origin are intersecting with one another in an angle of 45 degrees. When they intersect with the projecting plane what geometric figure do they produce? (1 points)

b) Show that the conic section $x_1^2 + x_1x_2 + x_3^2 - x_1x_3 - x_2x_3 - x_3^2 = 0$ is the same as the conic section with

$$C = \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

(Hint: do you know how to multiply matrices?) (4 points)

c) The distance between a 2D-point $x = (x_1, x_2)$ in Cartesian coordinates and a line $l = (a, b, c)$ can be computed using the distance formula

$$d = \frac{|ax_1 + bx_2 + c|}{\sqrt{a^2 + b^2}}$$

If two lines have value,

$$l_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad l_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

compute the distance between line l_1 and the intersection point of l_1 and l_2. Is it close to zero? Why/why not? (1 point)

Exercise 2: Projective transformation (14 points)

a) Show that $R^T R = I$ where R is the rotation matrix. Explain all your steps. (Hint: do you know how to transpose matrices?) (4 points)

b) If we have the projective transformation

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

- Compute the transformation $y_1 = Hx_1$ and $y_2 = Hx_2$ if

$$x_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

- Compute the lines l_1 containing x_1, x_2 and l_2 containing y_1, y_2
- Compute $(H^{-1})^T l_1$ and compare to l_2
- Show that projective transformations preserve lines. That is, for each line l_1 there is a corresponding line l_2 such that if x belongs to l_1 then the transformation $y = Hx$ belong to l_2. (Hint: If $l_1^T x = 0$ then $l_1^T H^-1 H x = 0$)

(4 points)

c) Given the line $x = (1, 3, 1)$, you want to rotate it of an angle $\alpha = \pi$ and transpose it of 2 both in x_1 and x_2 direction. Compute the new line equation. (6 points)