Assignment 5

Deadline: Tuesday, 20.12.2011

Assignment 5.1 (10 Points)

a) [5 Points] Explain the rationale behind the Fourier Series representation? For which type of functions is it defined and what is done? Which mathematical principle is used for computing the coefficients and why? Explain the relationship between the real Fourier Series representation and the complex Fourier Series representation?

b) [5 Points] Plot sin(kwx) and cos(kwx) for different k = 1,2,3,…and explain the mathematical concept of trigonometric orthogonality. Compute the integral

$$\int_{-\pi}^{\pi} \sin(kwx) \cdot \cos(lwx) dx$$

for w=k=1 and l=1.25. Use the resulting expression and the corresponding plot of the sin(kwx) and the cos(lwx) function to explain, why the trigonometric orthogonality property does only hold if both coefficients l and k are natural numbers.

Assignment 5.2 (10 Points)

a) [3 Points] Consider the complex Fourier Series Representation (FSR)

$$f(x) = \sum_{k \in \mathrm{Z}} c_k \cdot e^{ikwx}$$

and compute the limit of the FSR for an aperiodic function (T→∞), i.e. the limit

$$f(x) = \lim_{T \to \infty} \sum_{k \in \mathrm{Z}} c_k \cdot e^{ikwx}$$

What is the relation of this limit to the inverse Fourier Transform.

b) [7 Points] Compute the Fourier Transform of the following function f(x) manually (**be careful: partial integration**!!) and compare the plot of the original function f(x) with its Fourier spectrum F(w). Explain the meaning of w.

$$f(x) = \delta(x \in [0, \infty]) \cdot x$$