Image Analysis

7. Image Segmentation II

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1. Image Segmentation as Clustering
2. Multivariate Kernel Density Estimation
3. Mean Shift Segmentation

Neighborhoods

For a discrete image

$$f \in \mathbb{R}^{n \times m}$$

we call

$$I := \{1, \ldots, n\} \times \{1, \ldots, m\}$$

the **set of pixels** (also **grid**) and each $(x, y) \in I$ a **pixel**.

There are two different neighborhood systems in use:

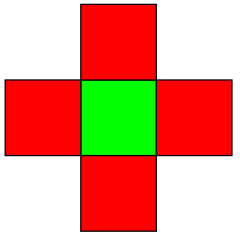
Two different pixels (x, y) and (x', y') are called **neighbors**

$$(x, y) \sim (x', y')$$

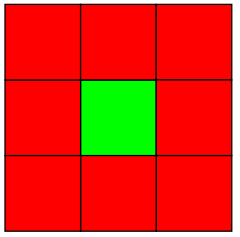
if

$(|x - x'| = 1$ and $y = y')$
or $(x = x'$ and $|y - y'| = 1)$

$|x - x'| \leq 1$ and $|y - y'| \leq 1$



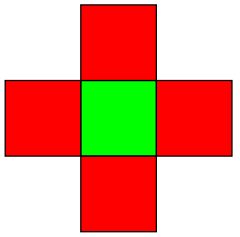
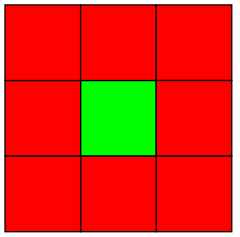
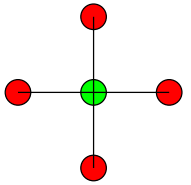
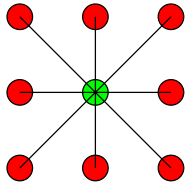
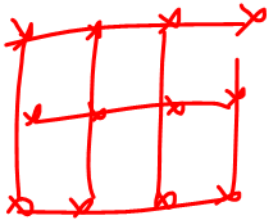
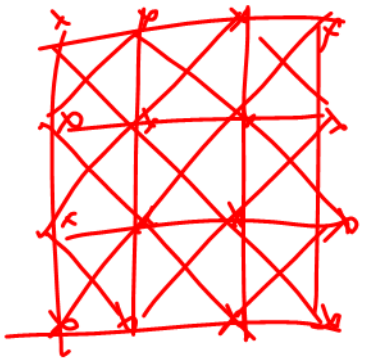
(**4-neighborhood**)



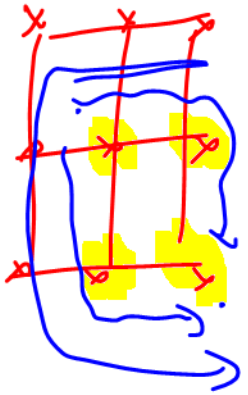
(**8-neighborhood**)

Neighborhoods (2/2)

The neighborhoods define the neighbor graph on the pixels I:

Regions



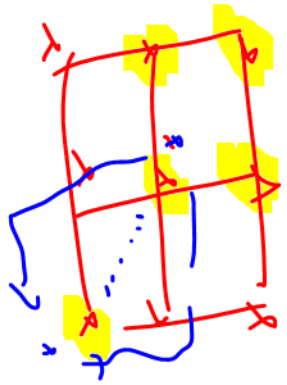
A **region** of an image $f \in \mathbb{R}^I$ is a subset of its pixels

$$R \subseteq I$$

that is connected w.r.t. the neighborhood graph.

This means: For each two pixels $(x, y), (x', y') \in R$ of the region there is a sequence

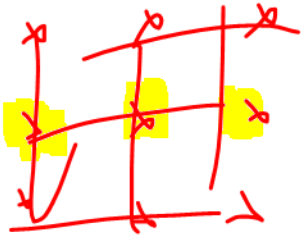
$$(x_1, y_1), \ldots, (x_T, y_T)$$



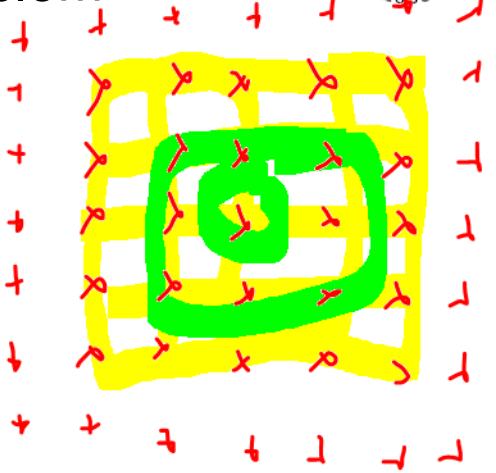
of pixels in R that

- starts in (x, y): $(x_1, y_1) = (x, y)$,
- ends in (x', y'): $(x_T, y_T) = (x', y')$,
- and where two consecutive pixels in the sequence are neighbors:

$$(x_t, y_t) \sim (x_{t+1}, y_{t+1}) \quad \forall t = 1, \ldots, T-1$$



The (Unsupervised) Image Segmentation Problem



Given an image $f \in \mathbb{R}^{n \times m}$, find regions

$$R_1, R_2, \ldots, R_k \subseteq \{1, \ldots, n\} \times \{1, \ldots, m\}$$

such that

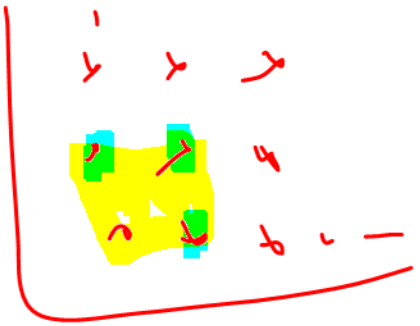
- the intensities in these regions

$$f_{|R_i}$$

are homogenuous, i.e., have similar values or vary only slowly,

and/or

- there are edges at the borders of the region.



The Supervised Image Segmentation Problem

Given a set of segmented images, i.e., images with attached segments

$$(f^1, s^1), (f^2, s^2), \ldots, (f^L, s^L)$$

with images $f^\ell \in \mathbb{R}^{I^\ell}$ and a set of segments

$$s^\ell = \{S_1^\ell, S_2^\ell, \ldots, S_{K^\ell}^\ell\}$$

with regions $S_k^\ell \subseteq I^\ell$ (for $\ell = 1, \ldots, L$, $k = 1, \ldots, K^\ell$).

Learn a **segmentation model**

$$\hat{s} : \bigcup_{n,m} \mathbb{R}^{n \times m} \to \mathcal{P}(\text{regions}(\mathbb{N} \times \mathbb{N}))$$

that assigns to each image f a set of regions $\hat{s}(f)$, s.t. for the given images as well as for new images (from the same distribution) a suitable error measure between the true segments s^ℓ and the predicted segments $\hat{s}(f^\ell)$ is minimal.

Image Segmentation as Clustering

The (unsupervised) image segmentation problem can be viewed as a clustering problem on the pixels of the image.

To do so, one has to describe pixels (x, y) by **pixel feature vectors** $\phi(x, y)$.

Simple feature vector: **pixel position**:

$$\phi(x, y) := (x, y)$$

This obviously does not describe pixels accurately as it does not depend on the intensities at all.

Another simple feature vector: **pixel intensity**:

$$\phi(x, y) := f(x, y)$$

This obviously does not describe pixels accurately as it does not depend on the pixel position, so it does not take any spatial relationships into account.

Pixel Features

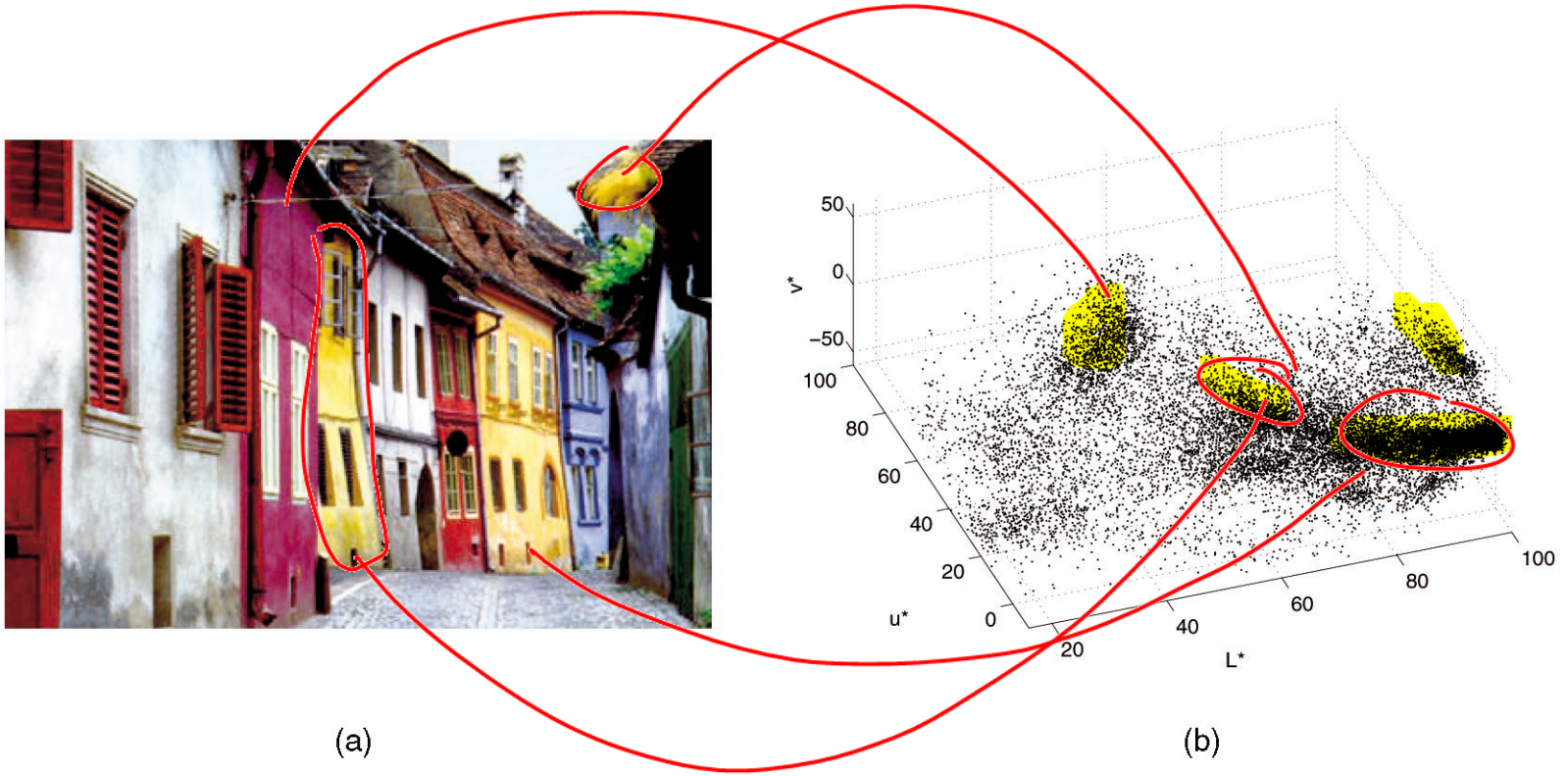
A simple useful pixel feature vector can be made from **a combination of position and intensity**:

$$\phi(x, y) := (x, y, f(x, y))$$

It also makes sense to include information about the intensities of neighborhood pixels.

Now any clustering algorithm can be applied to cluster the pixel feature vectors.

Pixel Features / Example



Example of a feature space. (a) A 400×276 color image. (b) Corresponding L*u*v* color space with $110,400$ data points.

(from [CM02])

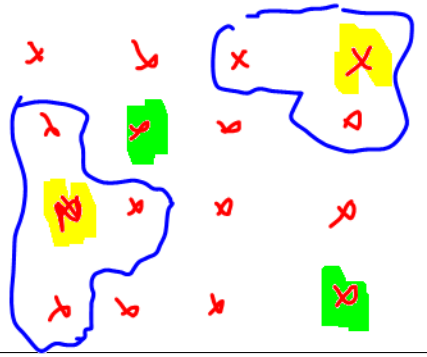
Distance-based Clustering

For example, with a simple distance measure on pixel features such as

$$d\left(\begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}, \begin{pmatrix} x' \\ y' \\ f(x',y') \end{pmatrix}\right) := \sqrt{(x-x')^2 + (y-y')^2 + \lambda(f(x,y) - f(x',y'))^2}$$

with a suitable weight $\lambda \in \mathbb{R}^+$, one can apply standard distance-based clustering algorithms such as **k-means** and **hierarchical clustering algorithms**.

This approach will join pixels in a cluster that are both, close to each other and have similar intensity values. But it does not guarantee that the clusters actually form regions.



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Multivariate Density Estimation / Empirical Distribution

Assume, we have N data points

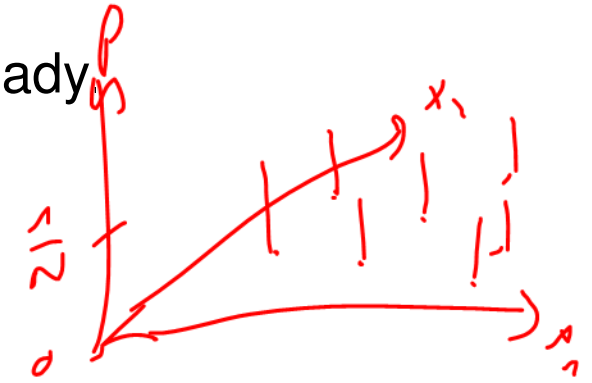
$$x_1, x_2, \ldots, x_N$$

in $\mathbb{R}^d$ and want to estimate the density $p(x)$ of their underlying distribution.

Simple estimator:

$$\hat{p}(x) := \begin{cases} \frac{1}{N}, & \text{if } x \in \{x_1, \ldots, x_N\} \\ 0, & \text{else} \end{cases}$$

- Assumes we have seen all possible points already
- Assumes all points are equally likely.
- In general, too close to training data.



Note: for the general case where there could be duplicate points, i.e., $i \neq j$ with $x_i = x_j$, the formula is

$$\hat{p}(x) := |\{x_i \,|\, x = x_i, i = 1, \ldots, N\}|/N$$

Multivariate Density Estimation / Uniform Distribution

Another simple estimator:

$$\hat{p}(x) := \begin{cases} \frac{1}{\text{vol}(R)}, & \text{if } x \in R \\ 0, & \text{else} \end{cases}$$

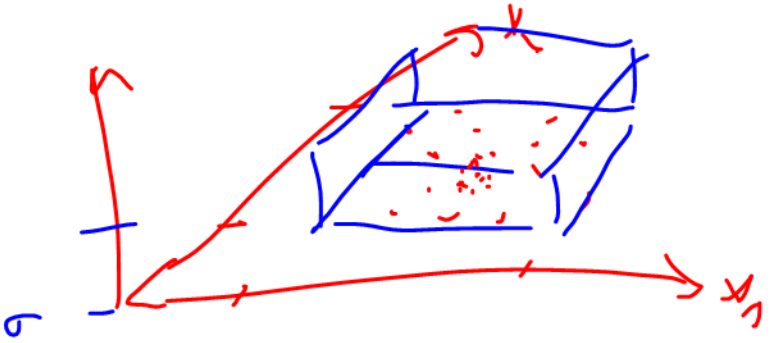
where $R \subseteq \mathbb{R}^d$ is a region that contains $x_1, \ldots, x_N$,
e.g.,

$$R := [\min_i x_{i,1}, \max_i x_{i,1}] \times \cdots \times [\min_i x_{i,d}, \max_i x_{i,d}]$$

$$\text{vol}(R) = (\max_i x_{i,1} - \min_i x_{i,1}) \cdots (\max_i x_{i,d} - \min_i x_{i,d})$$

– Assumes all points are equally likely.

– In general, does not take into account training data sufficiently, e.g., cannot distinguish between dense and sparse regions,



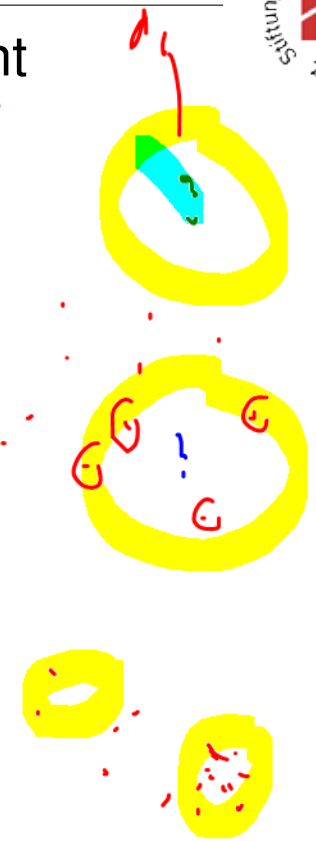
Multivariate Density Estimation / Frequency Count

A more appropriate estimator:

$$\hat{p}(x) := c \cdot |\{x_i \mid d(x, x_i) \leq d_0\}|$$

where

- $d : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ is a distance measure on $\mathbb{R}^d$,
- d_0 is a constant distant threshold and
- c is a constant that makes $\hat{p}$ to integrate to 1.



The frequeny count estimator

- can distinguish between dense and sparse regions,
- equals the empirical distribution for $d_0 = 0$,
- gets smoother for increasing d_0,
- is a step function.

The distance measure defines the shape of the neighborhood regions (euclidean $\rightsquigarrow$ ball, $L_1 \rightsquigarrow$ cube, etc.).

Multivariate Density Estimation / Parzen Window

To avoid the steps whenever an example enters or leaves the neighborhood region, one can weight the contribution of the points by their distance. Such a weight is called a kernel (aka similarity measure):

$$K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$$

Parzen window (aka **kernel density estimator**):

$$\hat{p}(x) := c\frac{1}{N}\sum_i K(x, x_i)$$

where

- c is a constant that makes $\hat{p}$ to integrate to 1.

The simplest kernel is the scalar product (called **linear kernel**):

$$K(x, y) := \langle x, y \rangle = \sum_{j=1}^{d} x_j y_j$$

Multivariate Kernel Density Estimation / Kernel Properties

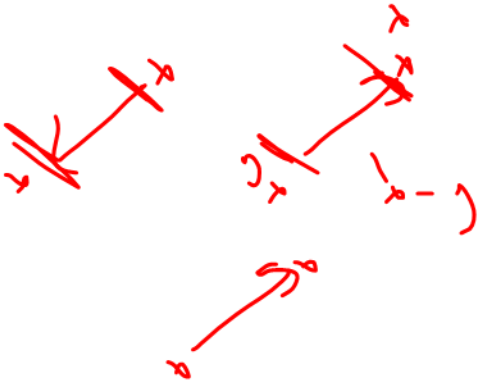
Most kernels are **shift-invariant**, i.e., they can be written as

$$K(x, y) = k(x - y)$$

for a kernel norm

$$k : \mathbb{R}^d \to \mathbb{R}$$

that is maximal at 0.



Many kernels are **radially symmetric**, i.e., they can be written as

$$K(x, y) = k(\langle x - y, x - y \rangle) = k(||x - y||^2)$$

with a 1-dimensional kernel norm $k : \mathbb{R}^+ \to \mathbb{R}$.

Often kernel norms k with unbounded support are **truncated**:

$$k^{\text{truncated}}(x) := \begin{cases} k(x), & \text{if } ||x|| < 1 \\ 0, & \text{else} \end{cases}$$

Multivariate Kernel Density Estimation / Kernel Properties (2/2)

For a kernel norm k, the maximal absolute radius with a non-vanishing weight

$$\text{bandwidth}(k) := \sup\{||x|| \mid x \in \mathbb{R}^d, k(x) \neq 0\}$$

is called **bandwidth of kernel norm k**.

If k is a kernel norm with bandwidth 1 and $h \in \mathbb{R}^+$, then

$$k'(x) := k(\frac{x}{h})$$

is a kernel norm with bandwidth h.

This way, kernels can be made more narrow or wider.

Multivariate Kernel Density Estimation / Kernel Examples

Examples:

Normal Kernel Norm:

$$k(x) := c\, e^{-\frac{1}{2}x^2}$$

Epanechnikov Kernel Norm:

$$k(x) := \begin{cases} c\,(1-x^2), & \text{if } |x| < 1 \\ 0, & \text{else} \end{cases}$$

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Density Gradient Decent

For clustering a set of points, one has to identify dense regions, i.e., local maxima (called **modes**) of the estimated density

$$\hat{p}(x) := c\frac{1}{N}\sum_i k(||x - x_i||^2)$$

For this, one looks for zeros of the density gradient :

$$\frac{\partial}{\partial x}\hat{p}(x) \stackrel{!}{=} 0$$

Density Gradient Decent

$$\begin{aligned}\frac{\partial}{\partial x}\hat{p}(x) &= c\frac{1}{N}\sum_i (\frac{\partial}{\partial x}k)(||x-x_i||^2)2(x-x_i) \\ &= 2c\frac{1}{N}\sum_i \alpha_i(x-x_i) \\ &= 2c\frac{1}{N}((\sum_i \alpha_i)x - \sum_i \alpha_i x_i) \stackrel{!}{=} 0\end{aligned}$$

with

$$\alpha_i := (\frac{\partial}{\partial x}k)(||x-x_i||^2)$$

and if $\sum_i \alpha_i \neq 0$ we get the gradient descent iteration

$$x^{(t+1)} := \frac{\sum_i x_i\,(\frac{\partial}{\partial x}k)(||x^{(t)}-x_i||^2)}{\sum_i (\frac{\partial}{\partial x}k)(||x^{(t)}-x_i||^2)}, \quad t = 0,1,2,\ldots$$

Density Gradient Decent

For the normal kernel $k(x) := e^{-\frac{1}{2}x^2}$, we have

$$\frac{\partial}{\partial x} k(x) = -e^{-\frac{1}{2}x^2} x$$

and thus $\sum_i \alpha_i < 0$.

For the Epanechnikov kernel

$$k(x) := \begin{cases} 1 - x^2, & \text{if } |x| < 1 \\ 0, & \text{else} \end{cases}$$

we get

$$\frac{\partial}{\partial x} k(x) = \begin{cases} -2x, & \text{if } |x| < 1 \\ 0, & \text{else} \end{cases}$$

and thus $\sum_i \alpha_i < 0$.

For the proof of convergence of the density gradient descent see [CM02].

Density Gradient Decent / Mode of attraction

All modes can be identified by starting from different starting positions x and following the gradient

$$x^{(0)} := x$$

$$x^{(t+1)} := \frac{\sum_i x_i \, (\frac{\partial}{\partial x} k)(||x^{(t)} - x_i||^2)}{\sum_i (\frac{\partial}{\partial x} k)(||x^{(t)} - x_i||^2)}, \quad t = 0, 1, 2, \ldots$$

The iteration stops when the difference between $x^{(t+1)}$ and $x^{(t)}$ becomes sufficient small.

The resulting local maximum $x^{(T)}$ is called **mode of attraction of** x:

$$\text{mode}(x) := x^{(T)}$$

A Kernel for Pixel Features

Pixel features such as

$$\phi(x, y) := (x, y, f(x, y))$$

consist of two components: the **spatial component**

$$(\phi(x, y))^s := (x, y)$$

and the **range component** (intensity component)

$$(\phi(x, y))^r := f(x, y)$$

While the spatial component has dimension 2, the range component can have any dimension p:

- $p = 1$ for grayscale images,
- $p = 3$ for RGB images,
- $p > 3$ for pixel features taking into account the neighborhood etc.

A Kernel for Pixel Features (2/2)

A suitable kernel for pixel features has to take into account the different scale niveaus and dimensions of the two feature components:

$$k'(x) := \frac{c}{h_s^2 \, h_r^p} \, k(||\frac{x^s}{h_s}||^2) \, k(||\frac{x^r}{h_r}||^2)$$

where

- x now denotes the pixel features ϕ, x^s and x^r its spatial and range component, respectively,
- k is a 1-dimensional kernel norm,
- h_s and h_r are the bandwidth of the spatial and range component,
- c is a constant that makes k' to integrate to 1,
- p is the dimension of the range features.

Mean Shift Segmentation

Mean shift segmentation with parameters h_s, h_r and M proceeds as follows:

1. For each pixel $x \in I$, compute its mode of attraction $\text{mode}(x)$

2. Group all pixels with the same mode of attraction in the same segment. So let

   $$z_1, z_2, \ldots, z_K$$

   be all modes, then form

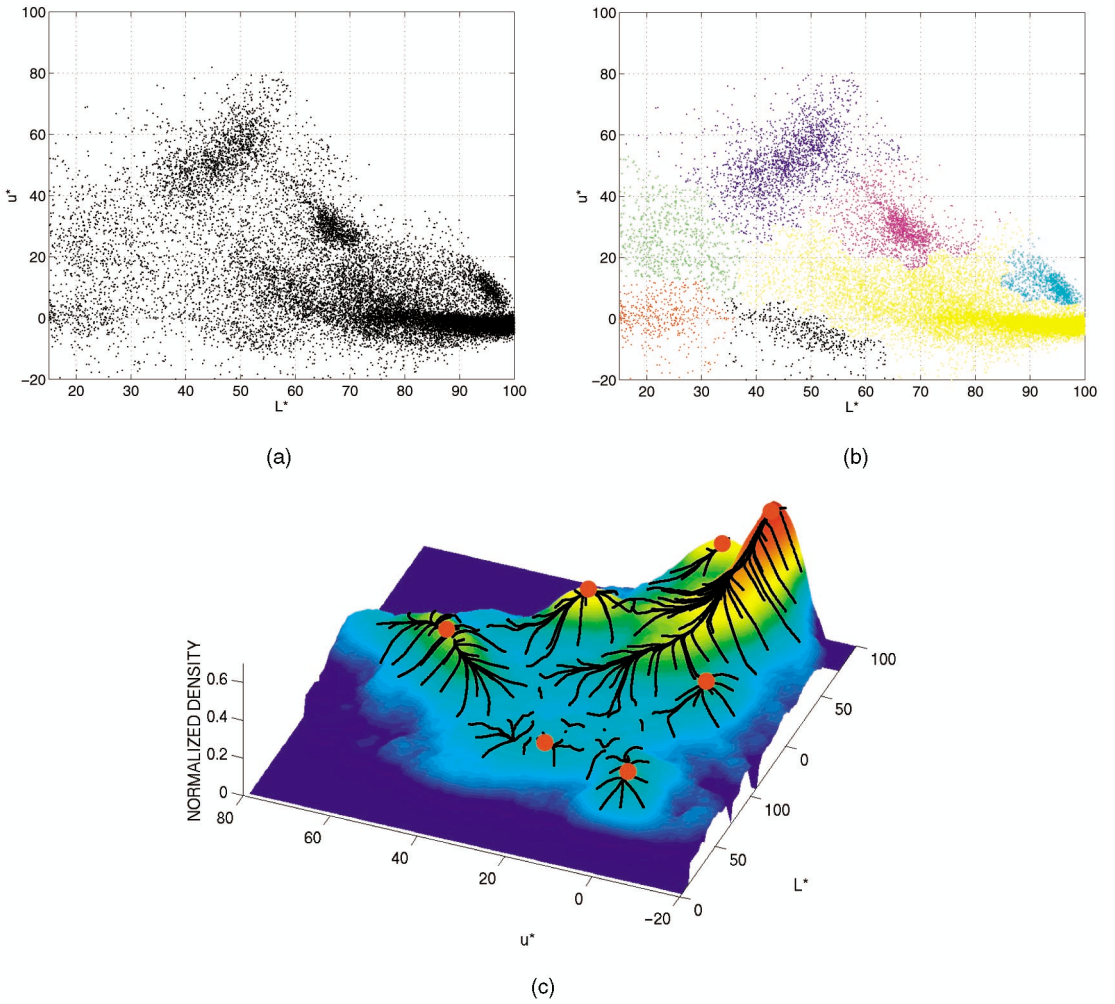
   $$S_k := \{x \in I \,|\, \text{mode}(x) = z_k\}, \quad k = 1, \ldots, K$$

   (called **basins of attraction**).

3. Iteratively join each two segments that contain two pixels with distance less than h_s and intensity difference of their modes of attraction less than h_r.

4. Eliminate segments having less than M pixels.

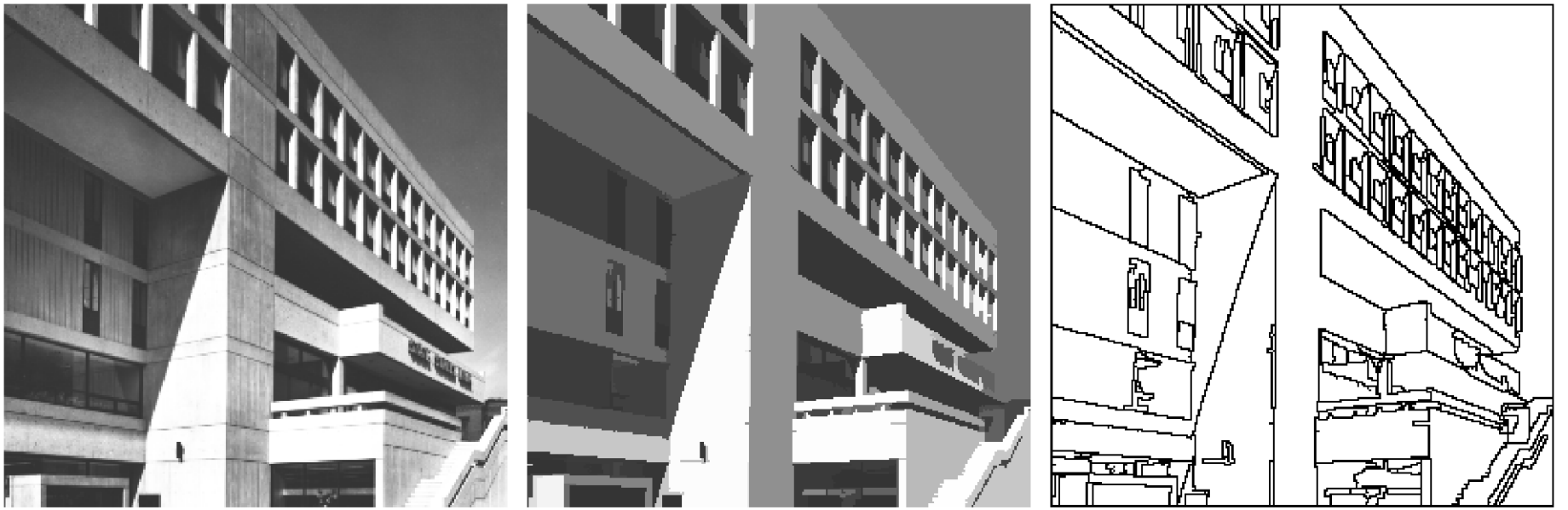
Mean Shift Segmentation



Example of a 2D feature space analysis. (a) Two-dimensional data set of $110,400$ points representing the first two components of the L*u*v*

(from [CM02])

Example

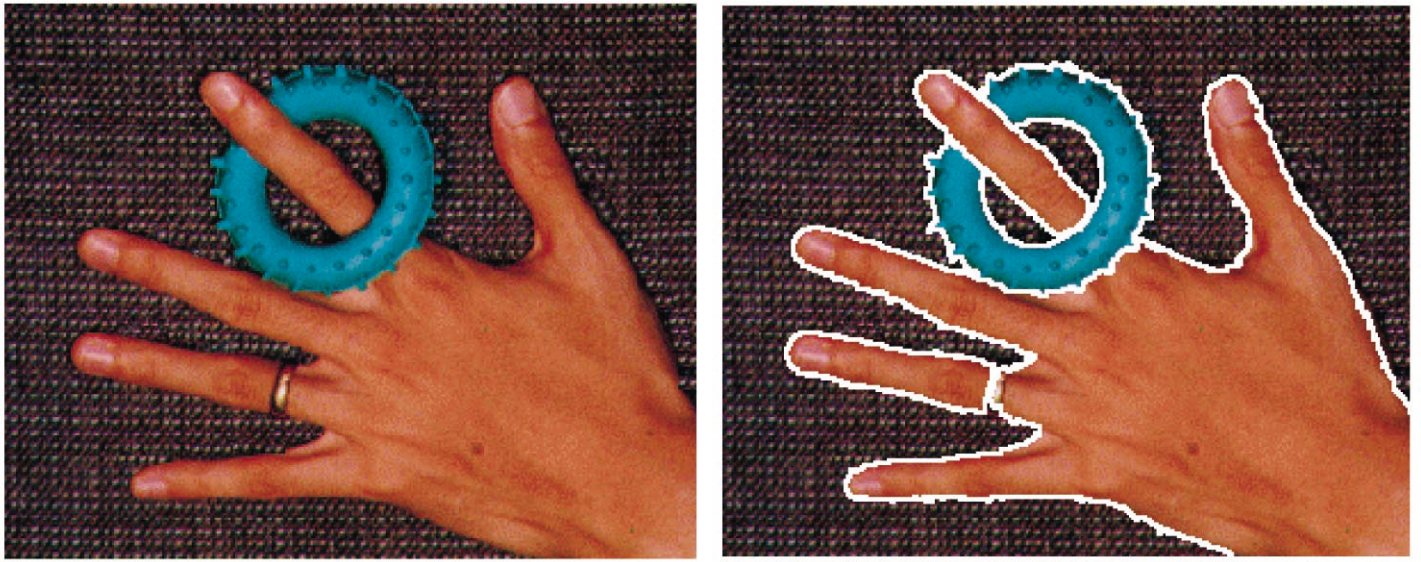


(a) (b) (c)

MIT image. (a) Original. (b) Segmented $(h_s, h_r, M) = (8, 7, 20)$. (c) Region boundaries.

(from [CM02])

Example



(a) (b)

Hand image. (a) Original. (b) Region boundaries delineated with $(h_s, h_r, M) = (16, 19, 40)$ drawn over the input.

(from [CM02])

Example

Landscape images. All the region boundaries were delineated with $(h_s, h_r, M) = (8, 7, 100)$ and are drawn over the original image.

(from [CM02])

Summary

- **(Unsupervised) Image Segmentation** is a clustering problem of the pixels of an image.
- For this one needs **pixel feature vectors**, e.g., a joint vector of pixel position and intensity.
- In principle, any clustering algorithm can be applied to the problem.
- **Mean shift segmentation** is a clustering procedure that (i) identifies the local maxima (modes) of an estimated density and (ii) assigns each point to a cluster described by its **mode of attraction**.
- Mean shift segmentation is a rather simple image segmentation procedure that has given quite reasonable results in practice.
- Besides unsupervised image segmentation, image segmentation also can be treated as **supervised problem** if already segmented images are available.

References

[CM02] Dorin Comaniciu and Peter Meer. Mean shift: A robust approach toward feature space analysis. *IEEE Trans. Pattern Anal. Machine Intell*, 24:603–619, 2002.