Machine Learning
Exercise Sheet 5

Prof. Dr. Dr. Lars Schmidt-Thieme, Martin Wistuba
Information Systems and Machine Learning Lab
University of Hildesheim

November 26th, 2013
Submission until December 3rd, 13.00 to wistuba@ismll.de

Exercise 11: IRLS (5 Points)

Given is following data:

y	x	y	x
0	9.5	1	11.1
0	9.6	1	11.1
0	9.7	1	11.1
0	9.8	1	11.5
0	9.9	1	11.8
0	10.5	1	11.9
0	11.0	1	12.1
0	11.2	1	12.2
0	11.5	1	12.5
0	11.7	1	12.6
0	12.1	1	12.6

a)

Apply linear regression for the target y using the method of R (see Exercise 4). Estimate the mean squared error of the model for the given data. Submit results and the code.

b)

Apply a logistic regression model for the target y. Use *iteratively reweighted least squares* for this and do it manually (for solving equation systems you can use a tool e.g. R). Stop after two iterations and estimate the mean squared error of the model on the given data. Compare the parameters after the first and second iteration with the final parameters $\hat{\beta}_0 = -23.35$, $\hat{\beta}_1 = 2.064$

c)

Plot the data and the predictor functions of (a) and (b). Discuss the results.

Exercise 12: Logistic Regression (5 Points)

The logistic regression model is defined as follows:

$$\log\left(\frac{p(Y=1|x,\beta)}{p(Y=0|x,\beta)}\right) = x^T\beta$$

a)

What is the difference between the linear and the logistic regression. Why is this extension/change useful?

b)

For estimating the probability $p(Y=1|x,\beta)$ the logistic regression model uses following equation:

$$p(Y=1|x,\beta) = \sigma(x^T\beta) = \frac{e^{x^T\beta}}{1+e^{x^T\beta}}$$

Show that

$$\frac{e^{x^T\beta}}{1+e^{x^T\beta}} = \frac{1}{1+e^{-x^T\beta}}$$

holds and that this prediction function follows directly from the model equation of the logistic model.

Hint: Consider the model equation of the logistic model and that $p(Y=1|x,\beta) + p(Y=0|x,\beta) = 1$.

c)

Prove the intermediate step in the prove of the IRLS method (Slide 18) following identity:

$$\frac{\partial\sigma(x^T\beta)}{\partial\beta} = \sigma(x^T\beta)(1-\sigma(x^T\beta))x$$

Hint: Use the fact that $\sigma(x^T\beta) = \frac{e^{x^T\beta}}{1+e^{x^T\beta}}$.