Machine Learning

C. Reinforcement Learning
C.2. Markov Decision Process

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University of Hildesheim, Germany

Outline

1. Introduction

2. Dynamic Programming and Reinforcement Learning

3. Monte Carlo Method

4. Temporal-Difference Prediction

5. Partially Observable Markov Decision Processes

Syllabus

Tue. 21.10.	(1)	0. Introduction
		A. Supervised Learning
Wed. 22.10.	(2)	A.1 Linear Regression
Tue. 28.10.	(3)	A.2 Linear Classification
Wed. 29.10.	(4)	A.3 Regularization
Tue. 4.11.	(5)	A.4 High-dimensional Data
Wed. 5.11.	(6)	A.5 Nearest-Neighbor Models
Tue. 11.11.	(7)	A.6 Decision Trees
Wed. 12.12.	(8)	A.7 Support Vector Machines
Tue. 18.11.	(9)	A.8 A First Look at Bayesian and Markov Networks
		B. Unsupervised Learning
Wed. 19.11.	(10)	B.1 Clustering
Tue. 25.11.	(11)	B.2 Dimensionality Reduction
Wed. 26.11.	(12)	B.3 Frequent Pattern Mining
		C. Reinforcement Learning
Tue. 2.12.	(13)	C.1 State Space Models
Wed. 3.12.	(14)	C.2 Markov Decision Processes

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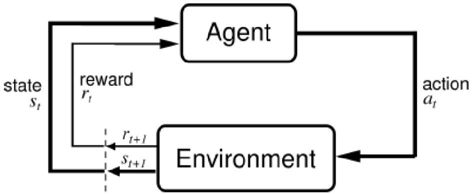

[SB98, fig. 3.1]

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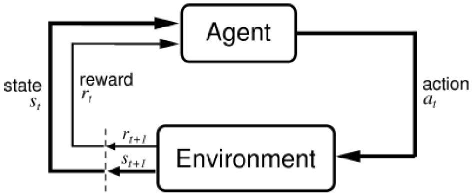

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[SB98, fig. 3.1]

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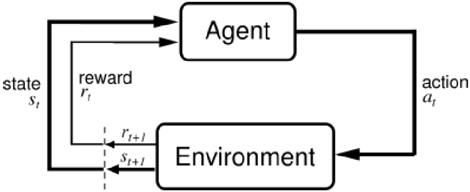

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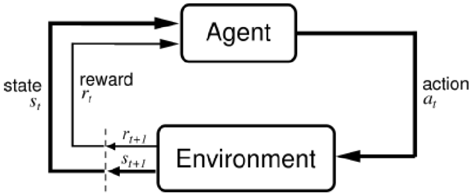

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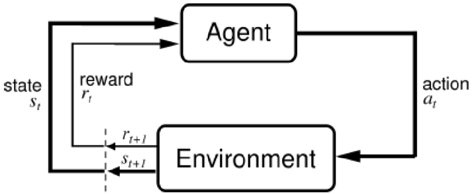

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- the agents tries to maximize the rewards over time

[SB98, fig. 3.1]

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- Our System is Markovian, so the transition function depends just on the current state:

$$P(s_{t+1} \mid s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(s_{t+1} \mid s_t, a_t) = T_{a_t}(s, s')$$

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- **Policy iteration** gives a greedy local search procedure based on the value of policies

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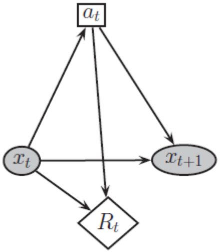

(b)

[Mur12, fig. 10.13 (b)]

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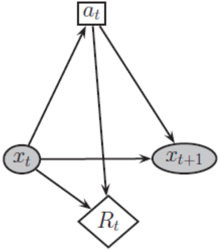

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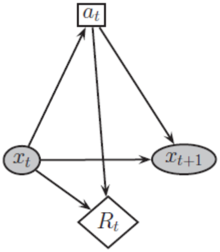

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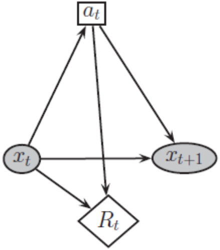

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- assume Markov Property holds as nearly as possible

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return means a function of future rewards that the agent seeks to maximize

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- assume the environment is a finite MDP, state and action sets are finite

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- both policy iteration and value iteration can be used

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- more efficient: instead of updating every state on every iteration, focus on **important states**

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  - useful when model is difficult to represent and/or learn

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  or: important states like: visited often during a game/procedure

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- only after completition of an episode value estimates and policies are changed

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$$V(s_t) \leftarrow V(s_t) + \alpha(R_t - V(s_t)), \alpha \in (0, 1) \text{ learning rate}$$

First-visit Monte Carlo policy evaluation

Initialize:

- $\pi \leftarrow$ policy to be evaluated
- $V \leftarrow$ an arbitrary state-value function
- $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Repeat forever:

- (a) Generate an episode using π
- (b) For each state s appearing in the episode:
  - $R \leftarrow$ return following the first occurrence of s
  - Append R to $Returns(s)$
  - $V(s) \leftarrow \text{average}(Returns(s))$

[SB98, fig. 5.1]

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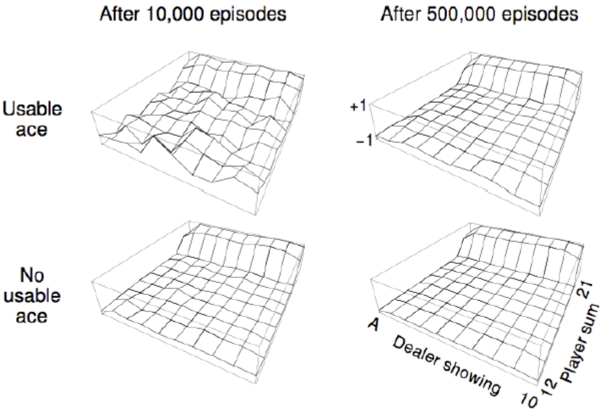
- **actions**: stick (stop receiving cards), hit (receive another card)
- **policy**: stick if sum is 20 or 21, else hit
- Dealer's fixed strategy: stick if ≤ 17 and hit if < 17

Example: Blackjack/approximate value functions



[SB98, fig. 5.2]

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2. Dynamic Programming and Reinforcement Learning

3. Monte Carlo Method

4. Temporal-Difference Prediction

5. Partially Observable Markov Decision Processes

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- if $V(s_{t+1})$ were correct, this would be like dynamic programming

TD Learning Algorithm

Initialize $V(s)$ arbitrarily, π to the policy to be evaluated
Repeat (for each episode):
  Initialize s
  Repeat (for each step of episode):
    $a \leftarrow$ action given by π for s
    Take action a; observe reward, r, and next state, s'
    $V(s) \leftarrow V(s) + \alpha\left[r + \gamma V(s') - V(s)\right]$
    $s \leftarrow s'$
  until s is terminal

[SB98, fig. 6.1]

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- both TD and MC converge, but TD usually learns faster

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- a probability distributions over state are called **belief states** b and the entire probability space the **belief space**
- so agent needs to update its belief upon taking action a and observation o, $b' = \tau(b, a, o)$, wth τ belief state transition function

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$$V^*(b) = \max_a \left[r(b,a) + \gamma \sum_{o \in O} \Omega(o \mid b, a) V^*(\tau(b,a,o)) \right]$$

Further Readings

- [SB98, chapter 5,6,7].
- [WvO12].

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