Machine Learning
Exercise Sheet 11

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Exercise 21: K-Means (10 Points)

Given is the following unlabeled data set:

x_1	x_2
2	2
2	1
1	1
0	0
-1	0
-2	0
-1	-1

Assume the first cluster center is given through

$$\mu_1 = \begin{pmatrix} 1 & 1 \end{pmatrix}^\top$$

a) For $K = 2$ and $K = 3$, compute the remaining cluster centers μ_2 and μ_3 (for $K = 3$) from the data set.

b) Perform one iteration (i.e. assigning instances to clusters using the given centers, then re-estimating the cluster centers) of K-Means for the given data and $K = 2$!

c) Perform a second iteration of K-Means for $K = 2$. Which instance is now differently clustered?

Exercise 22: Gaussian Mixture Models (10 Points)

A Gaussian mixture model containing $K = 3$ components has been learned for some one-dimensional training data. The individual Gaussians are given by

$$\mu_1 = -1 \qquad \mu_2 = 1 \qquad \mu_3 = 4$$

$$\sigma_1 = 1 \qquad \sigma_2 = 2 \qquad \sigma_3 = 0.4$$

Additionally, the probabilities for the individual clusters are:

$$\pi_1 = 0.4 \qquad \pi_2 = 0.4 \qquad \pi_3 = 0.2$$

a) Compute the responsibilities for a point $x \in \mathbb{R}$ to belong to a cluster i as:

$$r_i(x) = \frac{\pi_i \mathcal{N}(\mu_i, \sigma_i)}{\sum_{i'} \pi_{i'} \mathcal{N}(\mu_{i'}, \sigma_{i'})}$$

for all three clusters for the points $x \in \{-2, 2, 4.5, 6\}$ and assign the instances to clusters.

b) What happens if we extrapolate from the data, i.e. go to regions where we had no training data? Which Gaussian will be the dominant one? Explain why.

c) K-Means can be understood as a special form of GMM. Discuss, how this is possible, and describe the downsides of K-Means compared to GMMs.