Important information

- Deadline: Thursday Nov. 1, 2018 at 10:00 AM
- Fill out the tutorial group poll on learnweb[1]. The poll results will be published around We. Oct. 31, 12:00 AM.
- Put your printed or clearly handwritten results into the correct box at Samelsonsonplatz. (box number = 60 + group number). Write your name and matrikel number on the sheet.

Exercise 1 (Linear Regression Practice)**.**
Download the file 'tutorial3.dat' from the learnweb [2]. It contains two columns x, y of floating point data ($N = 50$).

1. (3p) Compute the least squares linear regression $\hat{y}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$ for the data.
2. (2p) A second linear model was fitted to the data by a different method. It resulted in the estimator $\hat{f}(x) = 1.4x$. Compute the mean square error (MSE) and the mean absolute error (MAE) both for your model and the proposed model.
$$\mathrm{MSE}(\hat{y}) = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}(x_i)|^2 \qquad \mathrm{MAE}(\hat{y}) = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}(x_i)|$$
3. (2p) Compute again the MSE and MAE of both estimators, but ignore the last 3 datapoints. What do you notice?
4. (3p) Plot the data and both functions. Which estimator would you prefer and why?

Exercise 2 (Linear Regression Theory)**.**
Given data $x = (x_i)_{i=1:N}$, $y = (y_i)_{i=1:N}$ you saw in the lecture that if we want to fit a linear model

$$\hat{y}(x) = \beta_0 + \beta_1 x$$

to the data, such that it minimizes the least squares error

$$(\hat{\beta}_0, \hat{\beta}_1) = \underset{(\beta_0,\beta_1)}{\operatorname{argmin}}\, \mathrm{MSE}(\hat{y}) \qquad \mathrm{MSE}(\hat{y}) = \frac{1}{N}\sum_{i=1}^{N} (y_i - (\beta_0 + \beta_1 x_i))^2$$

then the optimal parameters $(\hat{\beta}_0, \hat{\beta}_1)$ can be written in closed form as

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad \hat{\beta}_1 = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

[1] https://www.uni-hildesheim.de/learnweb2018/mod/ratingallocate/view.php?id=31928
[2] https://www.uni-hildesheim.de/learnweb2018/pluginfile.php/67442/mod_resource/content/1/tutorial3.dat

1. (3p) Show that $\hat{\beta}$ can be rewritten in vectorial form as

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad\qquad \hat{\beta}_1 = \frac{\frac{1}{N} x^\intercal y - \bar{x} \cdot \bar{y}}{\frac{1}{N} x^\intercal x - \bar{x}^2} \tag{1}$$

2. (2p) Show that the MSE can be rewritten in vectorial form as

$$\mathrm{MSE}(\hat{y}) = \frac{1}{N} \|y - A\beta\|_2^2$$

with

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \qquad A = \begin{bmatrix} 1 & x_1 \\ \vdots & \\ 1 & x_n \end{bmatrix} \qquad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

3. (3p) Show that

$$\frac{1}{2} \cdot \frac{\partial}{\partial \beta} \mathrm{MSE}(\hat{y}) = \begin{bmatrix} 1 & \bar{x} \\ \bar{x} & \frac{1}{N} x^\intercal x \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} - \begin{bmatrix} \bar{y} \\ \frac{1}{N} x^\intercal y \end{bmatrix}$$

4. (2p) Show that $\frac{\partial}{\partial \beta} \mathrm{MSE}(\hat{y}) = 0$ if and only if β satisfies the equations (1).