# Machine Learning

## B. Supervised Learning: Nonlinear Models
## B.3. Decision Trees

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# Syllabus

| | | |
|---|---|---|
| Fri. 26.10. | (1) | 0. Introduction |
| | | **A. Supervised Learning: Linear Models & Fundamentals** |
| Fri. 2.11. | (2) | A.1 Linear Regression |
| Fri. 9.11. | (3) | A.2 Linear Classification |
| Fri. 16.11. | (4) | A.3 Regularization |
| Fri. 23.11. | (5) | A.4 High-dimensional Data |
| | | **B. Supervised Learning: Nonlinear Models** |
| Fri. 30.11. | (6) | B.1 Nearest-Neighbor Models |
| Fri. 7.12. | (7) | B.2 Neural Networks |
| Fri. 14.12. | (8) | B.3 Decision Trees |
| Fri. 21.12. | (9) | B.4 Support Vector Machines |
| | | — *Christmas Break* — |
| Fri. 11.1. | (10) | B.5 A First Look at Bayesian and Markov Networks |
| | | **C. Unsupervised Learning** |
| Fri. 18.1. | (11) | C.1 Clustering |
| Fri. 25.1. | (12) | C.2 Dimensionality Reduction |
| Fri. 1.2. | (13) | C.3 Frequent Pattern Mining |
| Fri. 8.2. | (14) | Q&A |

# Outline


1. What is a Decision Tree?

2. Splits

3. Regularization

4. Learning Decision Trees

5. Split Quality Criteria

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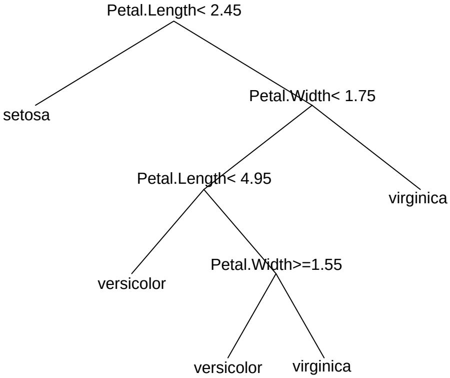
# Decision Tree

A **decision tree** is a tree that

1. at each **inner node** has a **splitting rule** that assigns instances uniquely to child nodes of the actual node, and
2. at each **leaf node** has a **prediction** (class label).

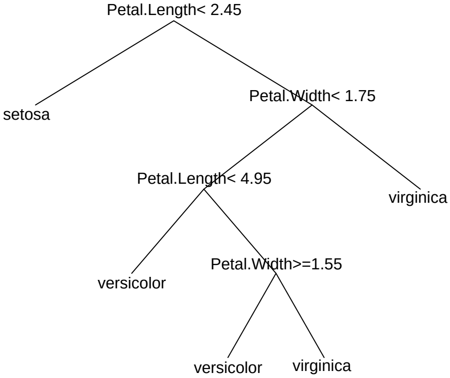



Note: The splitting rule is also called **decision rule**, the prediction the **decision**.

# Using a Decision Tree

The class of a given case $x \in \mathcal{X}$ is predicted by

1. starting at the root node,
2. at each interior node
   - evaluate the splitting rule for $x$ and
   - branch to the child node picked by the splitting rule, (default: left = "true", right = "false")
3. once a leaf node is reached,
   - predict the class assigned to that node as class of the case $x$.




Example:

x: Petal.Length = 6, Petal.Width = 1.6

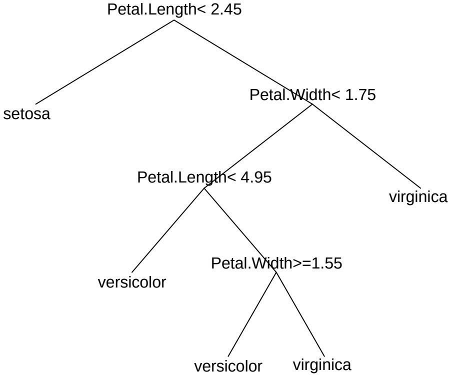
# Decision Tree as Set of Rules

Each branch of a decision tree can be formulated as a single conjunctive rule

*if* $condition_1(x)$ *and* $condition_2(x)$ *and* $\ldots$ *and* $condition_k(x)$,
*then* $y =$ *class label at the leaf of the branch.*

A decision tree is equivalent to a set of such rules,
one for each branch.

# Decision Tree as Set of Rules




set of rules:

Petal.Length $<$ 2.45 $\rightarrow$ class=setosa
Petal.Length $\geq$ 2.45 and Petal.Width $<$ 1.75 and Petal.Length $<$ 4.95 $\rightarrow$ class=versicolor
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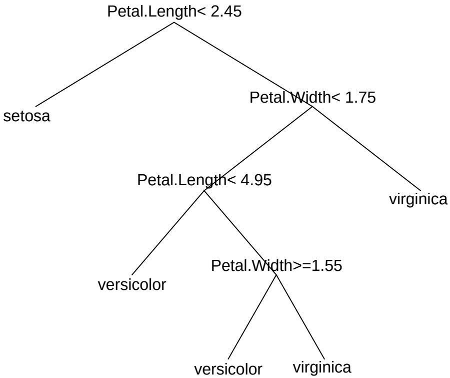
Petal.Length $\geq$ 2.45 and Petal.Width $\geq$ 1.75 $\rightarrow$ class=virginica

# Decision Tree as Set of Rules

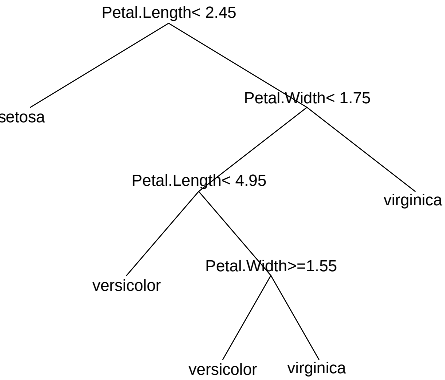


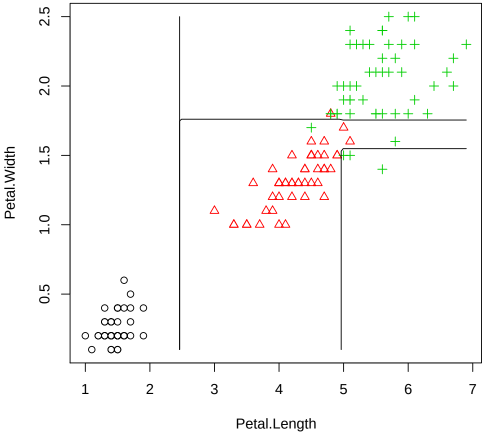

set of rules:

| | | |
|---|---|---|
| Petal.Length $< 2.45$ | | $\rightarrow$ class=setosa |
| Petal.Length $\in [2.45, 4.95[$ | and Petal.Width $< 1.75$ | $\rightarrow$ class=versicolor |
| Petal.Length $\geq 4.95$ | and Petal.Width $\in [1.55, 1.75[$ | $\rightarrow$ class=versicolor |
| Petal.Length $\geq 4.95$ | and Petal.Width $< 1.55$ | $\rightarrow$ class=virginica |
| Petal.Length $\geq 2.45$ | and Petal.Width $\geq 1.75$ | $\rightarrow$ class=virginica |

# Decision Boundaries

Decision boundaries are rectangular.



# Regression Tree

A **regression tree** is a tree that

1. at each **inner node** has a **splitting rule** that assigns instances uniquely to child nodes of the actual node, and
2. at each **leaf node** has a **target value**.

# Regression Tree & Probability Trees

A **regression tree** is a tree that

1. at each **inner node** has a **splitting rule** that assigns instances uniquely to child nodes of the actual node, and
2. at each **leaf node** has a **target value**.

A **probability tree** is a tree that

1. at each **inner node** has a **splitting rule** that assigns instances uniquely to child nodes of the actual node, and
2. at each **leaf node** has a **class probability distribution**.

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1. What is a Decision Tree?
2. Splits
3. Regularization
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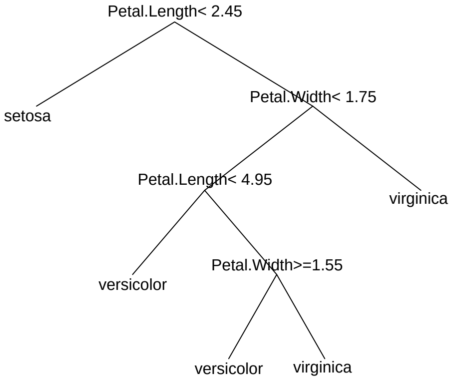
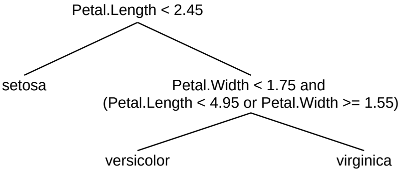
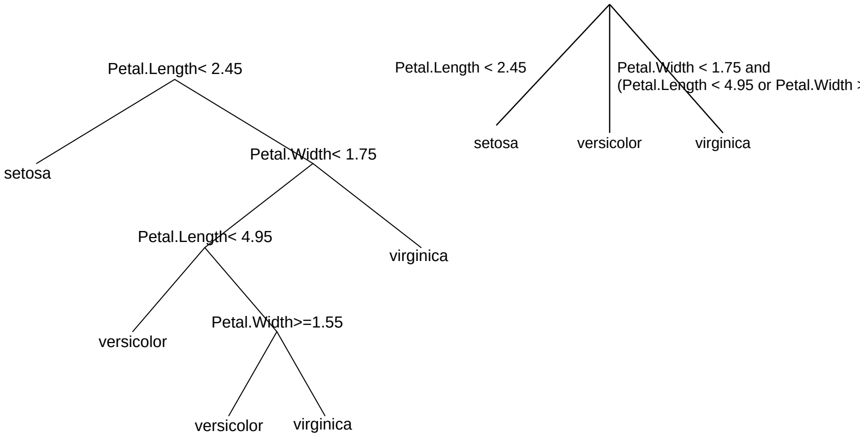
# An alternative Decision Tree?



# An alternative Decision Tree?



# Simple Splits

To allow all kinds of splitting rules at the interior nodes (also called **splits**) does not make much sense. The very idea of decision trees is that

- the **splits** at each node are rather **simple** and
- more complex structures are captured by **chaining several simple decisions** in a tree structure.

Therefore, the set of possible splits is kept small by opposing several types of restrictions on possible splits:

- by restricting the **number of variables** used per split (univariate vs. multivariate decision tree),
- by restricting the **number of children** per node (binary vs. n-ary decision tree),
- by allowing only some **special types** of splits (e.g., complete splits, interval splits, etc.).

# Types of Splits: Univarite vs. Multivariate

A split is called **univariate** if it uses only a single variable, otherwise **multivariate**.

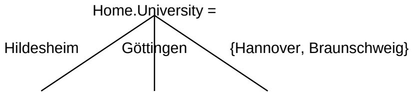
Example:
"Petal.Width $<$ 1.75" is univariate,
"Petal.Width $<$ 1.75 and Petal.Length $<$ 4.95" is bivariate.

- Multivariate splits that are mere conjunctions of univariate splits better would be represented in the tree structure.
- But there are also multivariate splits than cannot be represented by a conjunction of univariate splits, e.g.,
  "Petal.Width / Petal.Length $<$ 1"
  - can be represented by a univariate split on an additional predictor "Petal.Width / Petal.Length"

# Types of Splits: *n*-ary

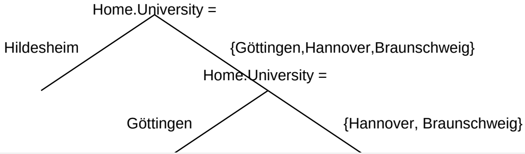
A split is called ***n*-ary** if it has *n* children.
(**Binary** is used for 2-ary, **ternary** for 3-ary.)

Example:
"Petal.Length $< 1.75$" is binary,



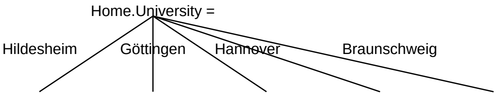

is ternary.

- All *n*-ary splits can be also represented as a tree of binary splits, e.g.,



# Types of Splits: Complete Splits

A univariate split on a nominal variable is called **complete**
if each value is mapped to a child of its own,
i.e., the mapping between values and children is bijective.




- A complete split is $n$-ary
  (where $n$ is the number of different values for the nominal variable).

# Types of Splits: Interval Splits

A univariate split on an at least ordinal variable is called **interval split** if for each child all the values assigned to that child are an interval.

Examples:
"Petal.Width $< 1.75$" is an interval split.

"(i) Petal.Width $< 1.45$,
(ii) Petal.Width $\geq 1.45$ and Petal.Width $< 1.75$,
(iii) Petal.Width $\geq 1.75$" also is an interval split.

"Petal.Width $< 1.75$ or Petal.Width $\geq 2.4$" is not an interval split.

# Types of Decision Trees

A decision tree is called

**univariate**,

$n$-**ary**,

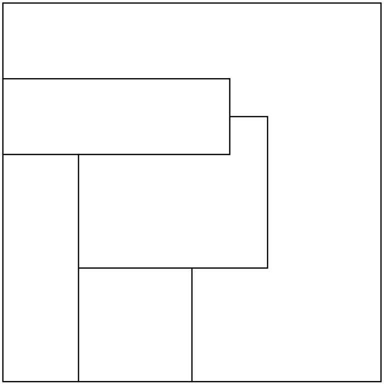
**with complete splits** or

**with interval splits**,

if **all** its splits have the corresponding property.

# Binary Univariate Interval Splits

There are partitions (sets of rules)
that cannot be created by binary univariate splits.



# Binary Univariate Interval Splits

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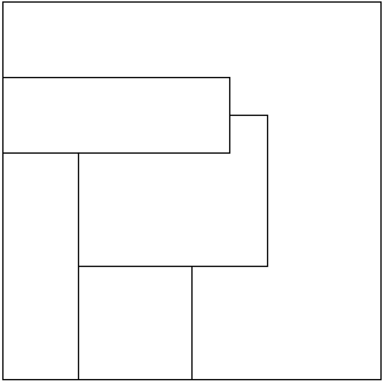

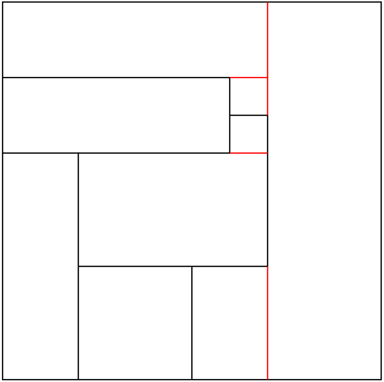

But all partitions can be refined
s.t. they can be created by binary univariate splits.

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# Learning Regression Trees (1/2)

Imagine, the **tree structure is already given**,
thus the partition

$$R_k, \quad k = 1, \ldots, K$$

of the predictor space is already given.

Then the remaining problem is to **assign a predicted value**

$$\hat{y}_k, \quad k = 1, \ldots K$$

to each cell.

# Learning Regression Trees (2/2)

Fit criteria such as the **smallest residual sum of squares** can be decomposed in partial criteria for cases falling in each cell:

$$\sum_{n=1}^{N}(y_n - \hat{y}(x_n))^2 = \sum_{k=1}^{K} \sum_{n=1, x_n \in R_k}^{n} (y_n - \hat{y}_k)^2$$

and this sum is minimal if the partial sum for each cell is minimal.

This is the same as fitting a constant model to the points in each cell and thus the $\hat{y}_j$ with smallest RSS are just the **means**:

$$\hat{y}_k := \text{average}\{y_n \mid n = 1, \ldots, N; x_n \in R_k\}$$

# Learning Decision Trees

The same argument shows that

- for a probability tree with given structure
  the **class probabilities with maximum likelihood** are just
  the **relative frequencies of the classes** of the points in that region:

$$\hat{p}(Y = y \mid x \in R_k) = \frac{|\{n \mid n = 1, \ldots, N; x_n \in R_k, y_n = y\}|}{|\{n \mid n = 1, \ldots, N; x_n \in R_k\}|}$$

- And for a decision tree with given structure, that
  the **class label with smallest misclassification rate** is just
  the **majority class label** of the points in that region:

$$\hat{y}(x \in R_k) = \arg\max_y |\{n \mid n = 1, \ldots, N; x_n \in R_k, y_n = y\}|$$

# Possible Tree Structures

- Even when possible splits are restricted,
  - e.g., only binary univariate interval splits are allowed,

  then tree structures can be built that separate all cases in tiny cells that contain just a single point
  (if there are no points with same predictors).

- For such a very fine-grained partition,
  the fit criteria would be optimal
  (RSS=0, misclassification rate=0, likelihood maximal).

- Thus, decision trees need some sort of regularization to make sense.

# Regularization Methods

There are several simple regularization methods:

**minimum number of points per cell**:

require that each cell (i.e., each leaf node) covers a given minimum number of training points.

**maximum number of cells**:

limit the maximum number of cells of the partition (i.e., leaf nodes).

**maximum depth**:

limit the maximum depth of the tree.

The number of points per cell, the number of cells, etc. can be seen as a **hyperparameter** of the decision tree learning method.

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# Decision Tree Learning Problem

The decision tree learning problem could be described as follows:
Given a dataset

$$\mathcal{D}^{\text{train}} := \{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$$

find a decision tree $\hat{y} : X \to Y$ that

- is binary, univariate, and with interval splits,
- contains at each leaf a given minimum number $m$ of examples,
- and has minimal misclassification rate

$$\text{mr}(\hat{y}; \mathcal{D}^{\text{train}}) := \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(y_n \neq \hat{y}(x_n))$$

among all those trees.

# Decision Tree Learning Problem

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among all those trees.

Unfortunately, this problem is **not feasible** as
there are **too many tree structures** / partitions to check
and no suitable optimization algorithms to sift efficiently through them.

# Greedy Search

Therefore, a greedy search is conducted that

- starting from the root
- builds the tree **recursively**
- by selecting the **locally optimal decision** in each step.
  - or alternatively, even just **some locally good** decision.

# Greedy Search / Possible Splits (1/2)

At each node one tries **all possible splits**.

For an univariate binary tree with interval splits
at the actual node let there still be the data

$$(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$$

Then check for each predictor variable $X$ with domain $\mathcal{X}$:

1. if $X$ is a **nominal variable**: (with $m$ levels)
   all $2^{m-1} - 1$ possible splits in two subsets $X_1 \dot{\cup} X_2$.

   E.g., for $\mathcal{X} = \{\text{Hi}, \text{Gö}, \text{H}\}$ the splits

   $$\begin{array}{lll} \{\text{Hi}\} & \text{vs.} & \{\text{Gö}, \text{H}\} \\ \{\text{Hi}, \text{Gö}\} & \text{vs.} & \{\text{H}\} \\ \{\text{Hi}, \text{H}\} & \text{vs.} & \{\text{Gö}\} \end{array}$$

# Greedy Search / Possible Splits (2/2)

2. if $X$ is an **ordinal** or **interval-scaled variable**:

sort the $x_n$ as

$$x'_1 < x'_2 < \ldots < x'_{n'}, \quad N' \leq N$$

and then test all $N' - 1$ possible splits at

$$\frac{x'_n + x'_{n+1}}{2}, \quad n = 1, \ldots, N' - 1$$

E.g.,

$$(x_1, x_2, \ldots, x_8) = (15, 10, 5, 15, 10, 10, 5, 5), \quad N = 8$$

are sorted as

$$x'_1 := 5 < x'_2 := 10 < x'_3 := 15, \quad N' = 3$$

and then split at 7.5 and 12.5.

# Greedy Search / Original Fit Criterion

All possible splits – often called **candidate splits** – are assessed by a **quality criterion**.

For all kinds of trees the **original fit criterion** can be used, i.e.,

for regression trees:

the **residual sum of squares**.

for decision trees:

the **misclassification rate**.

for probability trees:

the **likelihood**.

The split that gives the best improvement is chosen.

# Example

Artificial data about visitors of an online shop:

| | $x_1$ | $x_2$ | $x_3$ | $y$ |
|---|---|---|---|---|
| | referrer | num.visits | duration | buyer |
| 1 | search engine | several | 15 | yes |
| 2 | search engine | once | 10 | yes |
| 3 | other | several | 5 | yes |
| 4 | ad | once | 15 | yes |
| 5 | ad | once | 10 | no |
| 6 | other | once | 10 | no |
| 7 | other | once | 5 | no |
| 8 | ad | once | 5 | no |

Build a decision tree that tries to predict if a visitor will buy.

# Example / Root Split

**Step 1 (root node):** The root covers all 8 visitors.

There are the following splits:

| variable | values | buyer yes | buyer no | errors |
|---|---|---|---|---|
| referrer | {s} | 2 | 0 | 2 |
| | {a, o} | 2 | 4 | |
| referrer | {s, a} | 3 | 2 | 3 |
| | {o} | 1 | 2 | |
| referrer | {s, o} | 3 | 2 | 3 |
| | {a} | 1 | 2 | |
| num.visits | once | 2 | 4 | 2 |
| | several | 2 | 0 | |
| duration | $<7.5$ | 1 | 2 | 3 |
| | $\geq 7.5$ | 3 | 2 | |
| duration | $< 12.5$ | 2 | 4 | 2 |
| | $\geq 12.5$ | 2 | 0 | |

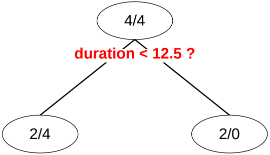
# Example / Root Split

The splits

- referrer = search engine ?
- num.visits = once ?
- duration $< 12.5$ ?

are locally optimal at the root.

We choose "duration $< 12.5$":




Note: See backup slides after the end for more examples.

# Decision Tree Learning Algorithm

expand−decision−tree(node $T$, training data $\mathcal{D}^{\text{train}}$):<br>
  if stopping−criterion ($\mathcal{D}^{\text{train}}$):<br>
    $T$.class $:= \arg\max_{y'} |\{(x,y) \in \mathcal{D}^{\text{train}} \mid y = y'\}|$<br>
    return<br>
  $s := \arg\max_{\text{split } s}$ quality-criterion($s$)<br>
  if $s$ does not improve:<br>
    $T$.class $= \arg\max_{y'} |\{(x,y) \in \mathcal{D}^{\text{train}} \mid y = y'\}|$<br>
    return<br>
  $T$.split $:= s$<br>
  for $z \in \text{Im}(s)$:<br>
    create new node $T'$<br>
    $T$.child[$z$] $:= T'$<br>
    expand−decision−tree($T'$, $\{(x,y) \in \mathcal{D}^{\text{train}} \mid s(x) = z\}$)

learn−decision−tree(training data $\mathcal{D}^{\text{train}}$):<br>
  create new node $T$<br>
  expand−decision−tree($T$, $\mathcal{D}^{\text{train}}$)<br>
  return $T$

# Decision Tree Learning Algorithm / Remarks (1/2)

- stopping-criterion($X$):
  e.g.,
  - all cases in $X$ belong to the same class,
  - all cases in $X$ have the same predictor values (for all variables),
  - there are less than the minimum number of cases per node to split.
- split $s$:
  all possible splits, e.g., all binary univariate interval splits.
- quality-criterion($s$):
  e.g., misclassification rate in $X$ after the split (i.e., if in each child node suggested by the split the majority class is predicted).

# Decision Tree Learning Algorithm / Remarks (2/2)

- $s$ does not improve:
  e.g., if the misclassification rate is the same as in the actual node (without the split $s$).
- $\text{Im}(s)$:
  all the possible outcomes of the split,
  e.g., $\{ 0, 1 \}$ for a binary split.
- $T.\text{child}[z] := T'$:
  keep an array that maps all the possible outcomes of the split to the corresponding child node.

# Decision Tree Prediction Algorithm

```
predict−decision−tree(node T, instance x ∈ ℝ^M):
   if T.split ≠ ∅:
      z := T.split(x)
      T′ := T.child[z]
   return predict-decision-tree(T′, x)
return T.class
```

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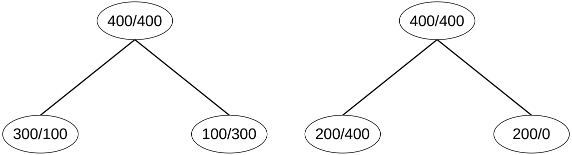
# Why Misclassification Rate is a Bad Split Quality Criterion

Although it is possible to use misclassification rate as quality criterion, it usually is not a good idea.

Imagine a dataset with a binary target variable (zero/one) and 400 cases per class (400/400).
Assume there are two splits:




Both have 200 errors / misclassification rate 0.25.

But the right split may be preferred as it contains a pure node.

# Split Contingency Tables

The effects of a split on training data can be described by a **contingency table** $(C_{j,k})_{j \in J, k \in K}$, i.e., a matrix

- with rows indexed by the different child nodes $j \in J$,
- with columns indexed by the different target classes $k \in K$,
- and cells $C_{j,k}$ containing the number of points in class $k$ that the split assigns to child $j$:

$$C_{j,k} := |\{(x, y) \in \mathcal{D}^{\text{train}} \mid s(x) = j \text{ and } y = k\}$$

# Entropy

Let

$$\Delta_n := \{(p_1, p_2, \ldots, p_N) \in [0,1]^N \mid \sum_n p_n = 1\}$$

be the set of multinomial probability distributions on the values $1, \ldots, N$.

An **entropy function** $q : \Delta_N \rightarrow \mathbb{R}_0^+$ has the properties

- $q$ is maximal for uniform $p = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$.
- $q$ is 0 iff $p$ is deterministic
  (one of the $p_n = 1$ and all the others equal 0).

# Entropy / Examples

**Cross-Entropy** / **Deviance**:

$$H(p_1, \ldots, p_N) := -\sum_{n=1}^{N} p_n \log(p_n)$$

**Shannons Entropy**:

$$H(p_1, \ldots, p_N) := -\sum_{i=1}^{n} p_n \log_2(p_n)$$

**Quadratic Entropy**:

$$H(p_1, \ldots, p_N) := \sum_{i=1}^{n} p_n(1 - p_n) = 1 - \sum_{n=1}^{N} p_n^2$$

Entropy measures can be extended to $\mathbb{R}_0^+$ via

$$q(x_1, \ldots, x_N) := q(\frac{x_1}{\sum_i x_i}, \frac{x_2}{\sum_n x_n}, \ldots, \frac{x_N}{\sum_n x_n})$$

# Entropy for Contingency Tables

For a contingency table $C_{j,k}$ we use the following abbreviations:

$$C_{j,.} := \sum_{k \in K} C_{j,k} \qquad \text{sum of row } j$$

$$C_{.,k} := \sum_{j \in J} C_{j,k} \qquad \text{sum of column } k$$

$$C_{.,.} := \sum_{j \in J} \sum_{k \in K} C_{j,k} \qquad \text{sum of matrix}$$

and define the following entropies:

**row entropy:**

$$H_J(C) := H(C_{j,.} \mid j \in J)$$

**column entropy:**

$$H_K(C) := H(C_{.,k} \mid k \in K)$$

**conditional column entropy:**

$$H_{K|J}(C) := \sum_{j \in J} \frac{C_{j,.}}{C_{.,.}} H(C_{j,k} \mid k \in K)$$

# Entropy for Contingency Tables

Suitable split quality criteria are

entropy gain:

$$HG(C) := H_K(C) - H_{K|J}(C)$$

entropy gain ratio:

$$HG(C) := \frac{H_K(C) - H_{K|J}(C)}{H_J(C)}$$

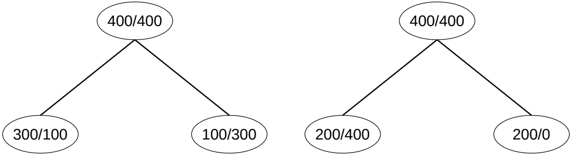
Shannon entropy gain is also called **information gain**:

$$\text{IG}(C) := -\sum_k \frac{C_{.,k}}{C_{.,.}} \log_2 \frac{C_{.,k}}{C_{.,.}} + \sum_j \frac{C_{j,.}}{C_{.,.}} \sum_k \frac{C_{j,k}}{C_{j,.}} \log_2 \frac{C_{j,k}}{C_{j,.}}$$

Quadratic entropy gain is also called **Gini index**:

$$\text{Gini}(C) := -\sum_k (\frac{C_{.,k}}{C_{.,.}})^2 + \underbrace{\sum_j \frac{C_{j,.}}{C_{.,.}} \sum_k (\frac{C_{j,k}}{C_{j,.}})^2}_{=:\text{gini-impurity}(C)}$$

# Entropy Measures as Split Quality Criterion




Both have 200 errors / misclassification rate 0.25.

But the right split may be preferred as it contains a pure node.

Gini-Impurity

$$= \frac{1}{2}((\frac{3}{4})^2 + (\frac{1}{4})^2) + \frac{1}{2}((\frac{3}{4})^2 + (\frac{1}{4})^2) = 0.625$$

Gini-Impurity

$$= \frac{3}{4}((\frac{1}{3})^2 + (\frac{2}{3})^2) + \frac{1}{4}(1^2 + 0^2) \approx 0.667$$

# Popular Decision Tree Configurations

| name | ChAID | CART | ID3 | C4.5 |
|---|---|---|---|---|
| author | Kass 1980 | Breiman et al. 1984 | Quinlan 1986 | Quinlan 1993 |
| selection measure | $\chi^2$ | Gini index, twoing index | information gain | information gain ratio |
| splits | all | binary nominal, binary quantitative, binary bivariate quantitative | complete | complete, binary nominal, binary quantitative |
| stopping criterion | $\chi^2$ independence test | minimum number of cases/node | $\chi^2$ independence test | lower bound on selection measure |
| pruning technique | none | error complexity pruning | pessimistic error pruning | pessimistic error pruning, error based pruning |

# Summary

- Decision trees are trees having
  - **splitting rules** at the **inner nodes** and
  - **predictions** (decisions) at the **leaves**.
- Decision trees use only **simple splits**
  - univariate, binary, **interval splits**.
- Decision trees have to be regularized by **constraining their structure**
  - minimum number of examples at inner nodes, maximum depth, etc.
- Decision trees are learned by greedy **recursive partitioning**.
  - As **split quality criteria** entropy measures are used
    - **Gini index**, **information gain ratio**, etc.
- Outlook (see lecture Machine Learning 2):
  - Sometimes **pruning** is used to make the search less greedy.
  - Decision trees use **surrogate splits** to cope with missing data.
  - Decision trees can be boosted yielding very competitive models (**random forests**, **gradient boosted decision trees**).

# Further Readings

- [Hastie et al., 2005, chapter 9.2+6+7], [Murphy, 2012, chapter 16.1–2], [James et al., 2013, chapter 8.1+3].

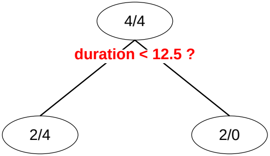
# References

Trevor Hastie, Robert Tibshirani, Jerome Friedman, and James Franklin. *The Elements of Statistical Learning: Data Mining, Inference and Prediction*, volume 27. Springer, 2005.

Gareth James, Daniela Witten, Trevor Hastie, and Robert Tibshirani. *An Introduction to Statistical Learning*. Springer, 2013.

Kevin P. Murphy. *Machine Learning: A Probabilistic Perspective*. The MIT Press, 2012.

# Example / Node 2 Split




The right node is pure and thus a leaf.

**Step 2 (node 2):** The left node (called "node 2") covers the following cases:

| | referrer | num.visits | duration | buyer |
|---|---|---|---|---|
| 2 | search engine | once | 10 | yes |
| 3 | other | several | 5 | yes |
| 5 | ad | once | 10 | no |
| 6 | other | once | 10 | no |
| 7 | other | once | 5 | no |
| 8 | ad | once | 5 | no |

# Example / Node 2 Split

At node 2 are the following splits:

| variable | values | buyer yes | buyer no | errors |
|---|---|---|---|---|
| referrer | {s} | 1 | 0 | 1 |
| | {a, o} | 1 | 4 | |
| referrer | {s, a} | 1 | 2 | 2 |
| | {o} | 1 | 2 | |
| referrer | {s, o} | 2 | 2 | 2 |
| | {a} | 0 | 2 | |
| num.visits | once | 1 | 4 | 1 |
| | several | 1 | 0 | |
| duration | $<7.5$ | 1 | 2 | 2 |
| | $\geq 7.5$ | 1 | 2 | |

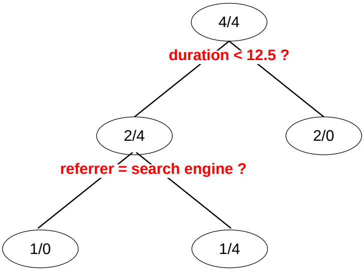
Again, the splits

- referrer = search engine ?
- num.visits = once ?

are locally optimal at node 2.

# Example / Node 5 Split

We choose the split "referrer = search engine":




The left node is pure and thus a leaf.

The right node (called "node 5") allows further splits.

# Example / Node 5 Split

**Step 3 (node 5):** The right node (called "node 5") covers the following cases:

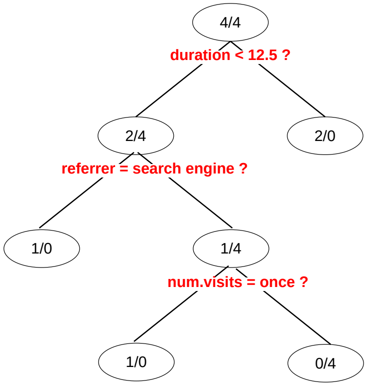
| | referrer | num.visits | duration | buyer |
|---|---|---|---|---|
| 3 | other | several | 5 | yes |
| 5 | ad | once | 10 | no |
| 6 | other | once | 10 | no |
| 7 | other | once | 5 | no |
| 8 | ad | once | 5 | no |

It allows the following splits:

| | | buyer | | |
|---|---|---|---|---|
| variable | values | yes | no | errors |
| referrer | {a} | 0 | 2 | 1 |
| | {o} | 1 | 2 | |
| num.visits | once | 1 | 0 | 0 |
| | several | 0 | 4 | |
| duration | $<7.5$ | 1 | 2 | 1 |
| | $\geq 7.5$ | 0 | 2 | |

# Example / Node 5 Split

The split “num.visits = once” is locally optimal.




Both child nodes are pure thus leaf nodes.
The algorithm stops.