

Deadline: Fr. January 15th, 10:00 Upload a .pdf file via LearnWeb. (e.g. exported Jupyter notebook)

1. SVM practice

(10 points)

A. [6p] To SVM in its dual form is written as the following optimization problem

$$\max_{\alpha} -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \langle x_i | x_j \rangle + \sum_{i=1}^n \alpha_i \quad \text{s.t.} \quad \begin{aligned} \sum_{i=1}^n \alpha_i y_i &= 0 \\ 0 &\leq \alpha_i \leq \gamma \end{aligned} \quad (1)$$

Rewrite the problem as an inequality constrained QPs, i.e. optimization problems of the form:

$$\min_z \frac{1}{2} z^T C z + c^T z \quad \text{s.t.} \quad \begin{aligned} A z &= a \\ B z &\leq b \end{aligned} \quad (2)$$

By explicitly constructing the matrices/vectors A, a, B, b, C, c . Note that z should be the concatenation of all variables.

B. [2p] Let $\gamma = 1$. Explicitly construct the matrices/vectors (A, a, B, b, C, c) of the **dual form** given the data from Table 1.

x_1	x_2	y	α_i
1	1	1	1
1	-1	-1	0.5
-1	1	-1	0.5
-1	-1	-1	0

Table 1: toy data

C. [2p] Classify the instance $(2, -2)^T$ using an SVM with the given α and $\beta_0 = -1$.

2. Constraint Optimization with Lagrange Multipliers

(10 points)

A. [5p] Minimize the function $f(x, y) = (x + y)^2$ subject to $-3x + y = -1$. What are all feasible minima?

B. [5p] Maximize the function $g(x) = 2x^3 - 3x$ subject to $x \leq 0.5$. What are all feasible maxima?