

# Machine Learning 2

## Exercise Sheet 2

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### Exercise 3: Gaussian Processes (5 Points)

For  $x \in [0, 5]$ , compute the Covariance function of a Gaussian Process using three different Kernels:

$$k_1(x_1, x_2) = \exp\left(-\frac{\|x_1 - x_2\|^2}{2}\right)$$

$$k_2(x_1, x_2) = (x_1^\top x_2 + 2)^2$$

$$k_3(x_1, x_2) = \exp(-|x_1 - x_2|)$$

Then, sample a set of 10 different latent functions  $f$  according to the Gaussian process prior:

$$f \sim \mathcal{N}(0, K)$$

for all three different Kernel/Covariance Matrices and plot them. What are the differences?

### Exercise 4: Gaussian Process Regression (5 Points)

Given are two training examples

$$X = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

with ground truth given as:

$$y = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Learn a Gaussian Process on the model and predict the mean and average for the instance  $x = 0$ . Use the squared exponential kernel

$$k(x_1, x_2) = \sigma_f^2 \exp\left(-\frac{\|x_1 - x_2\|^2}{2l^2}\right)$$

with  $l = 1$ ,  $\sigma_f = 1$ . Plot the model and add the standard deviation.

The underlying data generation function is  $f(x) = x^2$  and therefore the true label for  $x = 0$  is 0. Relearn the model including the new data instance and plot it again.