

Modern Optimization Techniques - Exercise Sheet 3

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Solutions need to be handed in until **Monday, November 17th, 2014 at 12:00**

Exercise 1: Linear Regression with Gradient Descent (13P)

We want to learn a non-regularized linear regression model using gradient descent on the data given by the design matrix X and labels y :

$$X = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad y = \begin{pmatrix} 11 \\ 10 \\ 8 \end{pmatrix}$$

Therefore, we need to find the parameter vector $\beta = (\beta_0, \beta_1, \beta_2)$ that minimizes the loss over all instances x_i :

$$\mathcal{L}(X, \beta, y) = \sum_{i=1}^3 (\beta^\top x_i - y_i)^2$$

- a) Explain in your own words, why we apply an approximate learning algorithm to a problem where an analytical solution exists?
- b) Compute the closed form solution for a non-regularized linear regression optimized for least squares!
- c) Assume your model is initialized by $\beta = (1, 1, 1)$, compute the errors

$$e_i = \beta^\top x_i - y_i$$

and the overall loss of the model.

- d) Using the error terms, compute the updates of β with a step size of $\mu = 0.1$. What are the errors and the overall loss after updating once?

Exercise 2: Backtracking Line Search (7P)

Let us define a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ through:

$$f(x_1, x_2) = x_1^2 + x_2^2$$

- a) Suppose you want to do a backtracking line search using the negative gradient $\Delta x = -\nabla f(x)$ as descent direction. Suppose you are in a current point $x' = (x'_1, x'_2)$, write down the backtracking condition

$$f(x + \mu \Delta x) > f(x) + a\mu \nabla f(x) \Delta x$$

for these special settings.

- b) We pick $a = 0.5$, $b = 0.1$ and start with a rather high initial step size $\mu = 10$. How small does μ have to become for the backtracking condition to be false? How many backtracking iterations will be done until this happens?