Modern Optimization Techniques - Exercise Sheet 2

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Solutions need to be handed in until Tuesday, November 3rd, 2015 at 10:00

Exercise 1: Convex Sets (10P)

- a) Explain in your own words what a convex set is and why it is needed to define convex functions.
- b) Let

$$D = \{ x \in \mathbb{R}^2 \mid 0 \le x_1 \le 2 \land 0 \le x_2 \le 3 \}$$

be a subset of \mathbb{R}^2 . Draw a scetch of D. Show, by using the definition, that D is a convex set.

c) Let A and B be two overlapping convex sets in \mathbb{R}^2 . Proove that their intersection

 $C=A\cap B$

is also a convex set.

Hint: Assume the opposite and show how this creates a contradiction

Exercise 2: Second Order Convexity Condition (10P)

A function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is convex if and only if:

- dom f is convex
- $\nabla^2 f(x) \succeq 0$ (The Hessian Matrix is positive semidifinite)

a) Draw a one dimensional function $(f : \mathbb{R} \longrightarrow \mathbb{R})$ and explain why this makes sense

Show that the following functions are convex by using the second order convexity condition:

- b) $f_1 : \mathbb{R} \longrightarrow \mathbb{R}^+ f_1(x) = e^{ax} \quad a \in \mathbb{R}$
- c) $f_2 : \mathbb{R} \longrightarrow \mathbb{R} f_2(x) = x \cdot \log x$
- d) $f_3 : \mathbb{R}^2 \longrightarrow \mathbb{R} f_3(x_1, x_2) = x_1^2 + x_2^2$