Modern Optimization Techniques - Exercise Sheet 10

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Solutions need to be handed in until **Thursday, February 2nd, 2017 at 12:00**

Exercise 1: Barrier Methods in General (10P)

a) Explain in your own words how the barrier method works for a general constrained minimization problem!

b) Consider the simple optimization problem:

$$\begin{aligned} \text{minimize} \quad & x^2 + 1 \\ \text{subject to} \quad & x \in [2, 4] \end{aligned}$$

Plot $tf_0(x) + \phi(x)$ for values of $t = 1, 5, 10, 20$. Label $x^\star(t)$ in your plots. Does it converge (in the sense of t) to the minimizer of the original problem?

c) Derive the dual feasible pair $\lambda^\star(t), \nu^\star(t)$ if instead $h(u) = -1/u$ is used in the definition of barrier function

$$\phi(x) = \sum_{i=1}^{m} h(f_i(x))$$

Exercise 2: Applying the Barrier Method (10P)

What happens if the barrier method is applied to the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & x_2 \\ \text{subject to} \quad & x_1 \leq x_2 \\ & 0 \leq x_2 \end{aligned}$$

Formulate the optimization problem into the standard form to then write down the derived optimization problem. Solve it by using a Newton step as centering step. Start in the feasible point $x = (-0.5, 1)$, use a step size of $\mu = 0.05$ and $t = 10$. Plot the iterates and explain what is happening.