Modern Optimization Techniques - Exercise Sheet 6

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December 8, 2016

Solutions need to be handed in via **Learnweb** until **Thursday, December 15th, 2016 at 12:00 pm**

Exercise 1: Subgradients and Subdifferentials (12P)

For the following functions, create a plot of them and find the points x_0 where they are not differentiable. Compute all the subdifferentials $\partial f(x_0)$ in the respective points!

a)

$$f(x) = \max\left\{0, \frac{1}{2}(x^2 - 1)\right\}$$

b)

$$f(x) = \begin{cases} -3x - 2 & x \in (-\infty, -1] \\ x^2 & x \in (-1, 2) \\ 5x - 6 & x \in [2, \infty) \end{cases}$$

Exercise 2: L1-Norm (8P)

Let us consider the $\ell 1$ Norm, which is defined by:

$$\|x\|_1 := \sum_{i=1}^{n} |x_i|$$

Compute the subdifferential of $\|\cdot\|_1$! Use the fact, that $\|\cdot\|_1$ can be written as maximum of 2^n linear functions

$$\|x\|_1 = \max\left\{s^\top x \,|\, s_i \in \{-1, 1\}\right\}.$$