Modern Optimization Techniques - Exercise Sheet 8

Lydia Voß

voss@ismll.de

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Solutions need to be handed in until **Thursday, January 19th, 2017 at 12:00**

Exercise 1: Constrained Minimization (12P)

For the two following constrained problems, plot the level sets of f_0 and the given constraints to then graphically find $x^\star$.
a)

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to} \quad & h(x_1, x_2) = x_1 + 2x_2 = 3 \end{aligned}$$

Write down the KKT conditions for this optimization problem and analytically compute $x^\star$!

b)

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = x_1 + x_2 \\ \text{subject to} \quad & h(x_1, x_2) = x_1 - x_2 = 2 \\ & f_1(x_1, x_2) = x_1 \geq 0 \\ & f_2(x_1, x_2) = x_2 \geq 0 \end{aligned}$$

Reason why you cannot compute the dual problem for a linear program as this one!

Exercise 2: Newton Algorithm for Equality Constrained Problems (8P)

Let us again consider the following equality constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to} \quad & h(x_1, x_2) = x_1 + 2x_2 = 3 \end{aligned}$$

Optimize this problem using the Newton Algorithm for Equality Constrained Problems with a step size of $\mu = 1$. Start it once in the feasible point $x = (0, 1.5)$ and once in the non-feasible point $x = (0, -5)$. How many iterations does the algorithm need to converge? Explain your findings!