Modern Optimization Techniques - Exercise Sheet 10

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Solutions need to be handed in until **Monday, January 29th, 2018 at 10:00 am**

Exercise 1: Active set Method (12P)

Given the following Quadratic Programming problem:

$$\begin{aligned} \text{minimize} \quad & f_0(x_1, x_2) = \frac{1}{2} x^T P x + q^T x \\ \text{suject to} \quad & 2x_1 + x_2 \leq 2 \\ & x_1 - x_2 \leq 1 \\ & -x_1 - x_2 \leq 1 \\ & -2x_1 + x_2 \leq 2 \end{aligned}$$

With

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \qquad q = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$$

Draw the feasible region and do the active set method with the initial point $x_0 = (-1, 0)^T$ with the so called working set $\mathcal{Q} = \{3, 4\}$.

Hint:First compute $\tilde{g}$ you can use $\tilde{g} = P * x^{(k)} + q$. Then compute d as the solution of the quadratic programming problem below. λ are the Lagrangian multipliers. After the 1. iteration move until constraint 1 becomes active.

For solving the quadratic programming with respect to the working set

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2} d^T G d + \tilde{g}^T d \\ \text{subject to} \quad & Ad = 0 \end{aligned}$$

to find the descent direction you can use a program or

$$\begin{pmatrix} P & A^T \\ A & 0 \end{pmatrix} * \begin{pmatrix} d^* \\ \nu^* \end{pmatrix} = \begin{pmatrix} -\tilde{g} \\ 0 \end{pmatrix}$$

Exercise 2: Gradient projection Method (8P)

Let us consider the following constrained optimization problem

$$
\begin{aligned}
\text{minimize} \quad & f_0(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 - 2x_1 - 3x_4 \\
\text{subject to} \quad & 2x_1 + x_2 + x_3 + 4x_4 = 7 \\
& x_1 + x_2 + 2x_3 + x_4 = 6 \\
& x_i \geq 0 \quad i = 1, 2, 3, 4
\end{aligned}
$$

Compute the direction of the projective negative gradient. Start in the feasible point $x = (2, 2, 1, 0)^T$.