Information Systems 2

5. Business Process Modelling I: Models

Lars Schmidt-Thieme

Information Systems and Machine Learning Lab (ISMLL)
Institute for Business Economics and Information Systems
& Institute for Computer Science
University of Hildesheim
http://www.ismll.uni-hildesheim.de

1. **Petri Nets**

2. **The Pi Calculus**

Overview

- Petri nets are models for parallel computation.

- A Petri net represents a parallel system as graph of component states (places) and transitions between them.

- You can executew Petri nets online (jPNS) at

  http://robotics.ee.uwa.edu.au/pns/java/

  There also is a more advanced open source Petri net editor (PIPE2):

  http://pipe2.sourceforge.net/

- Petri Nets have been invented by the German mathematician Carl Adam Petri in 1962.

Definition

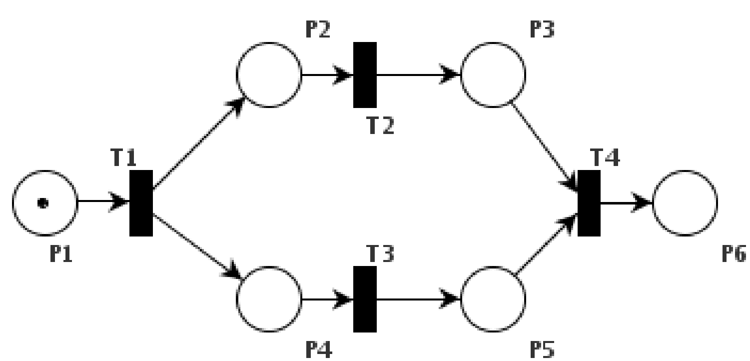
A **Petri net** is a directed graph $(P \dot\cup T, F)$ over two (disjoint) sorts of nodes, called **places** P and **transitions** T respectively, where

– all roots and leaves are places and

– edges connect only places with transitions,
and not places with places or transitions with transitions,
i.e., $F \subseteq (P \times T) \cup (T \times P)$.

Graphical representation:

places	—	circles
transitions	—	bars (or boxes)

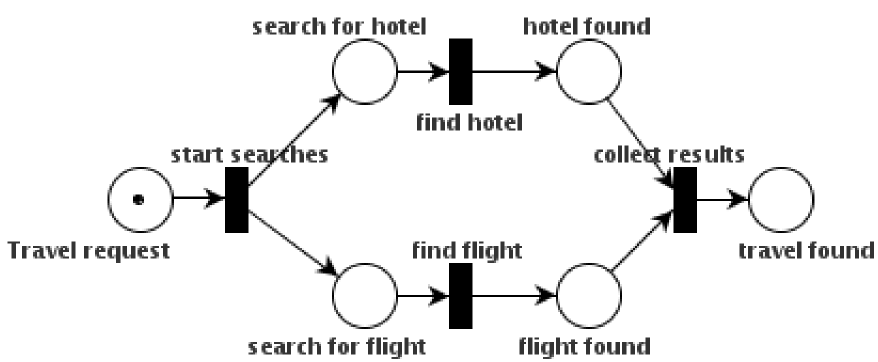


Interpretation

The components of a Petri net have the following interpretation:

– places denote a **stopping point in a process** as, e.g., the attainment of a milestone;
from the perspective of a transition, a place denotes a **condition**.

– transitions denote an **event** or **action**.



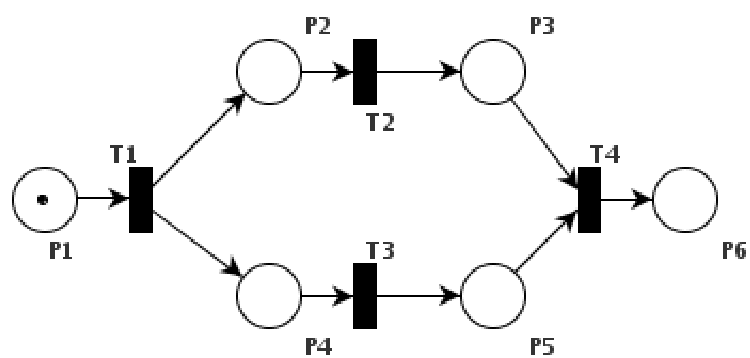
Inputs and Outputs

inputs / **preconditions** of a transition $t \in T$:
the places with edges into t, i.e.,

$$\bullet t := \text{fanin}(t) := \{p \in P \,|\, (p,t) \in F\}$$

outputs / **postconditions** of a transition $t \in T$:
the places with edges from t, i.e.,

$$t\bullet := \text{fanout}(t) := \{p \in P \,|\, (t,p) \in F\}$$



State of a Petri Net

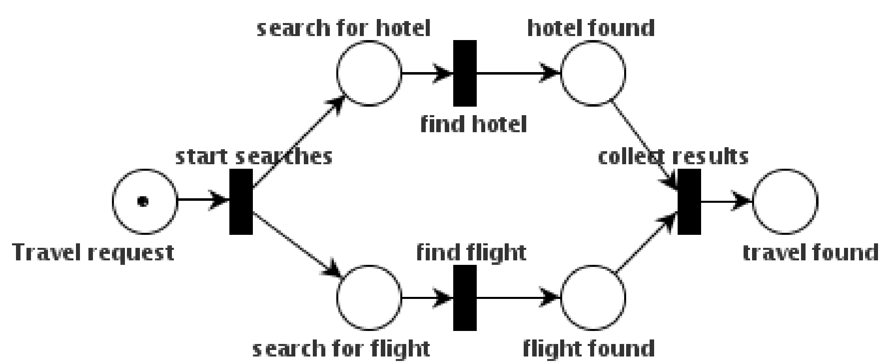
The state of a Petri net is described by the **markings** of the places by **tokens**, i.e.,

$$M : P \to \mathbb{N}$$

where $M(p)$ denotes the number of tokens assigned to place $p \in P$ at a given point in time.

Graphical representation:

tokens — black dots



State Change of a Petri Net

A transition $t \in T$ is said to be **enabled** if each of its inputs contains at least one token, i.e.,

$$M(p) \geq 1 \quad \forall p \in \bullet t$$

An enabled transition $t \in T$ may **fire**, i.e., change the state of the Petri net from a state M into a new state M^{new} by

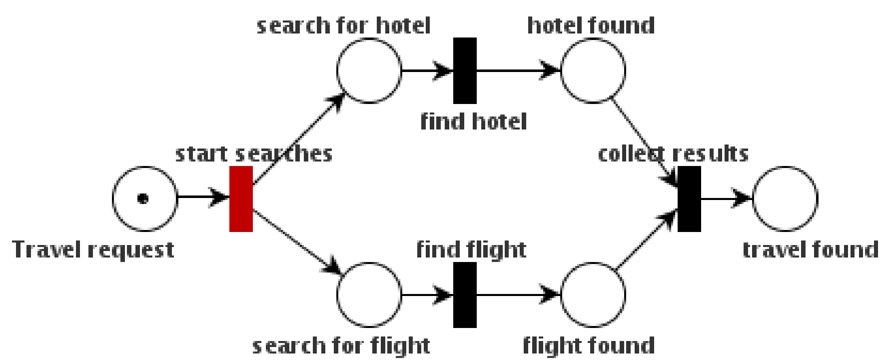
- remove one token from each of its inputs and
- add one token to each of its outputs, i.e.,

$$M^{\text{new}}(p) := M(p) - 1 \quad \forall p \in \bullet t,$$
$$M^{\text{new}}(p) := M(p) + 1 \quad \forall p \in t \bullet$$

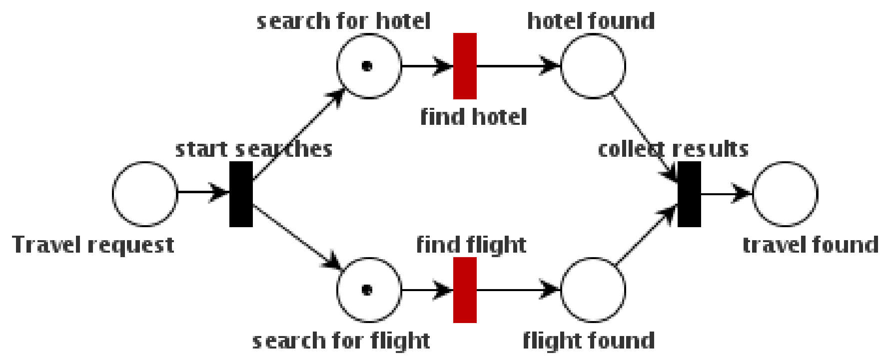
The new state is also denoted by $t(M) := M^{\text{new}}$.

If several transitions are enabled, the next transition to fire is choosen at random.

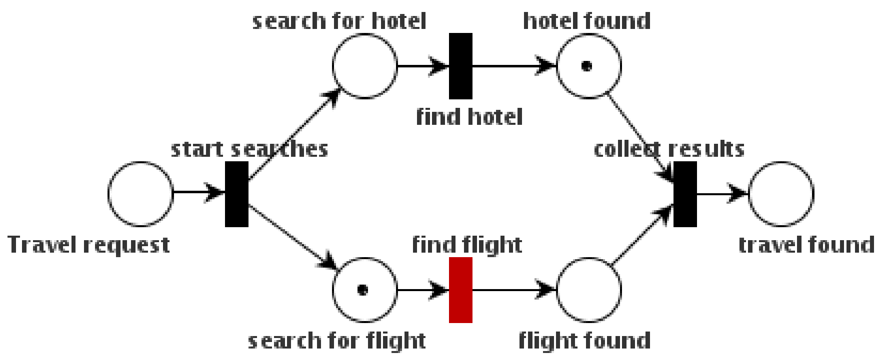
State Change of a Petri Net / Example



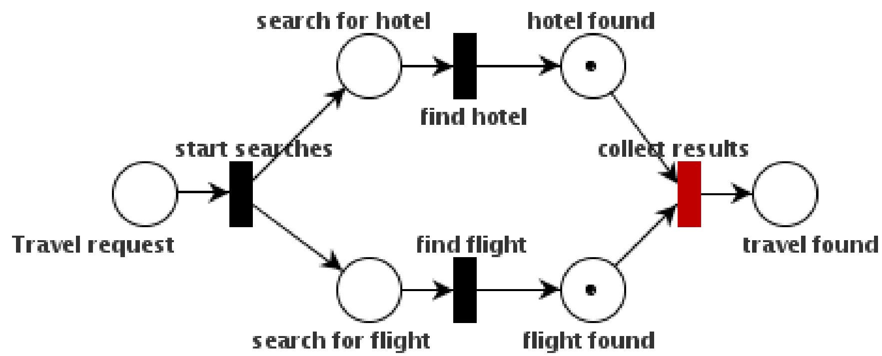
State Change of a Petri Net / Example



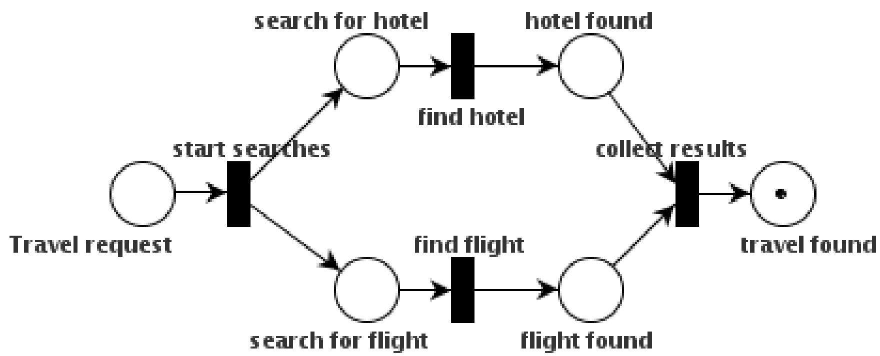
State Change of a Petri Net / Example



State Change of a Petri Net / Example

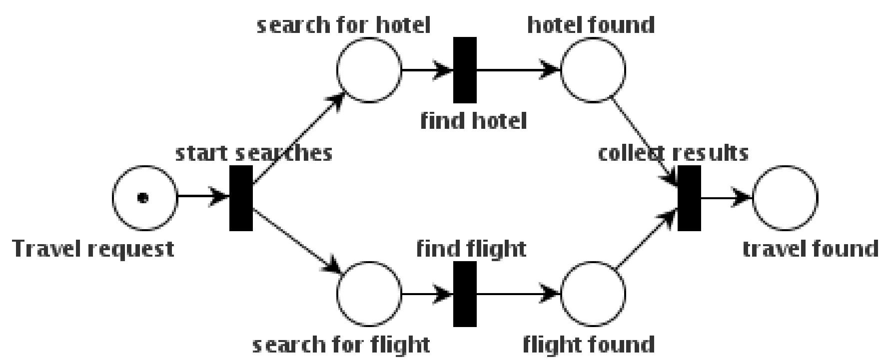


State Change of a Petri Net / Example

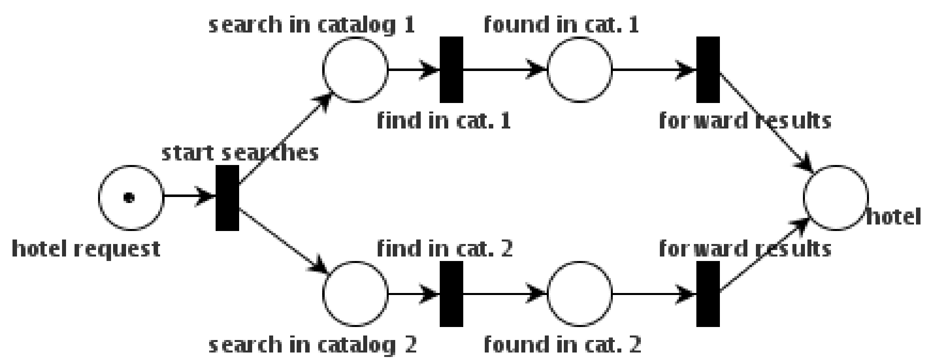


AND vs. OR

AND: both inputs are required.

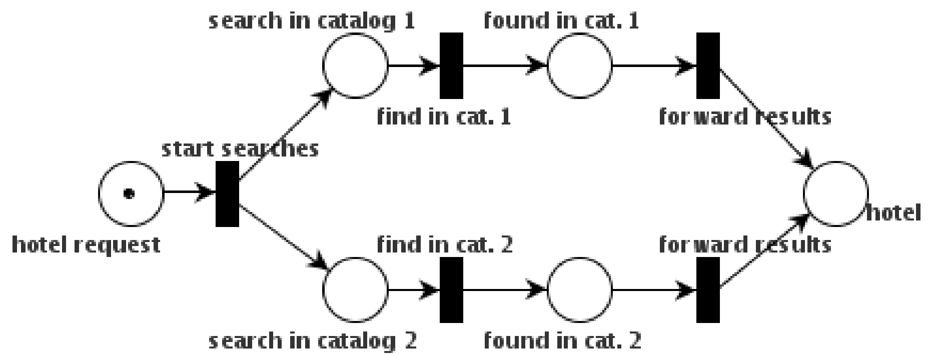


OR: at least one input is required.

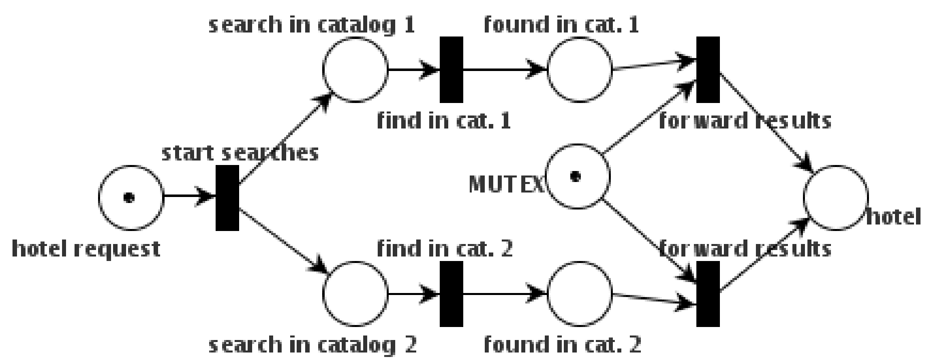


OR vs. XOR

OR: at least one input is required.



XOR: exactly one input is required.



Example (1/4)

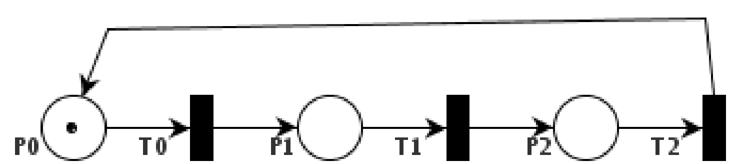
Assume there is a robot with three states:

- P0 robot works outside special workplace
- P1 robot waits for access to special workplace
- P2 robot works inside special workplace

and three events:

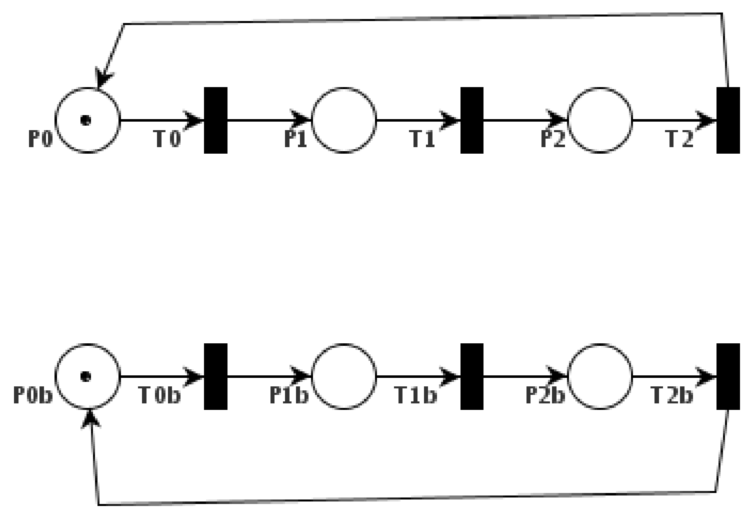
- T0 finish work outside special workplace
- T1 enter special workplace
- T2 finish work in special workplace

that works repeatedly:



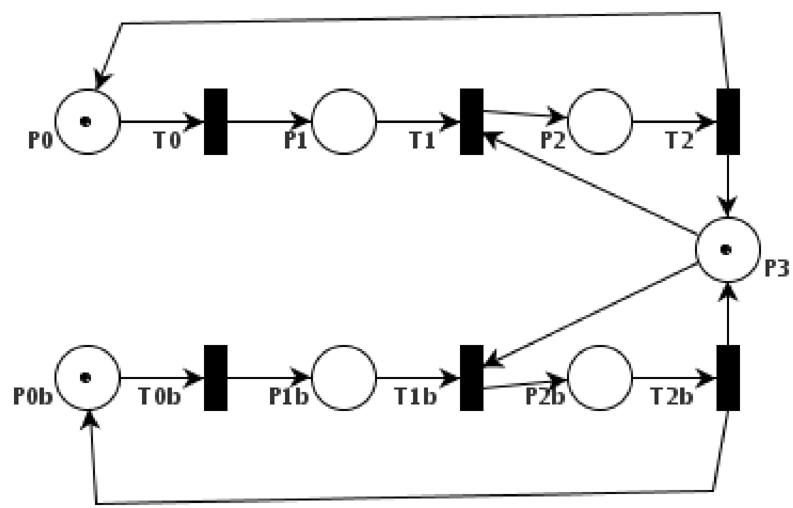
Example (2/4)

A system consisting of two such robots can be described as follows:



Example (3/4)

Now assume the special workplace cannot be used by both robots at the same time:

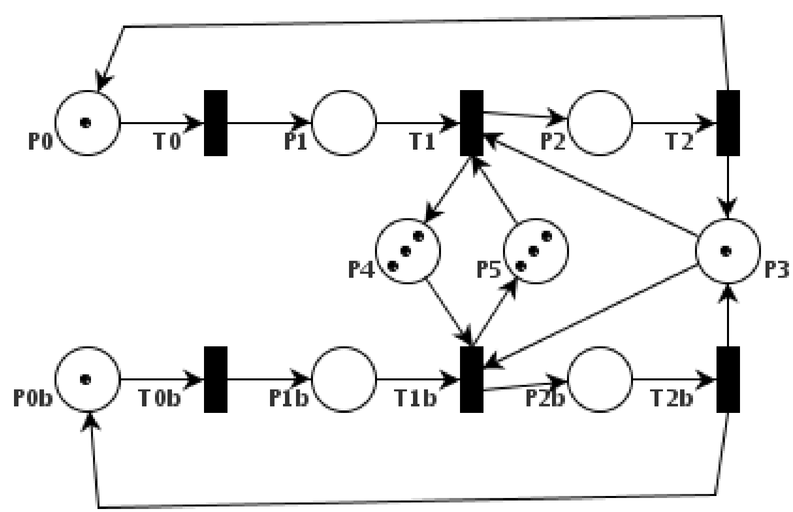


with additional place:

P3 special workplace available

Example (4/4)

Now assume a third robot assembles one component produced by the two robots each immediately and its input buffer can hold maximal 4 components.



with additional places:

P4 buffer place for component 2 available
P5 buffer place for component 1 available

Reachability

A given marking N of a Petri net is said to be **reachable from a marking M** if there exist transistions $t_1, t_2, \ldots, t_n \in T$ with

$$N = t_n(t_{n-1}(\ldots t_2(t_1(M))\ldots))$$

Example:
1. The state

$$P0 = 0, P1 = 0, P2 = 1, P0b = 1, P1b = 0, P2b = 0, P3 = 0$$

denoting the first robot to work in the special workplace while the second works outside, is reachable for the net robots (3/4) from the initial marking

$$P0 = 1, P1 = 0, P2 = 0, P0b = 1, P1b = 0, P2b = 0, P3 = 1$$

by the transition sequence $T0, T1$.

2. The state

$$P0 = 0, P1 = 0, P2 = 1, P0b = 0, P1b = 0, P2b = 1, P3 = 0$$

denoting both robots to work in the special workplace, is not reachable from the initial state.

Boundedness and Saveness

For $k \in \mathbb{N}$, a Petri net is called **k-bounded for an initial marking M** if no state with a place containing more than k tokens is reachable from M.

A Petri net is called **save for an initial marking M**, if it is 1-bounded for M.

Boundedness and Saveness / Example (1/2)

Two robots with states:

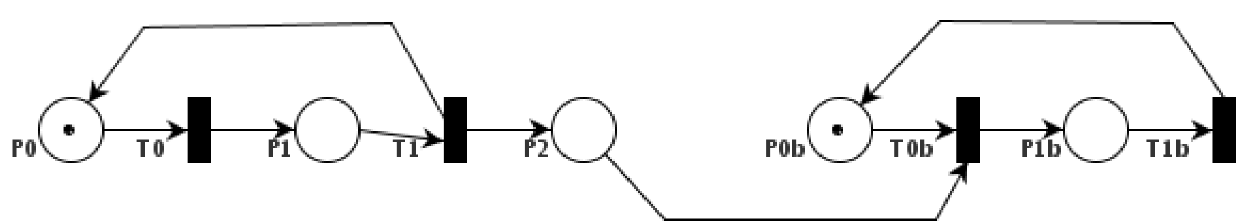
- P0 robot available
- P1 robot works on component

and events:

- T0 start working
- T1 finish work on component

working in sequence.

- P2 input component for second robot available



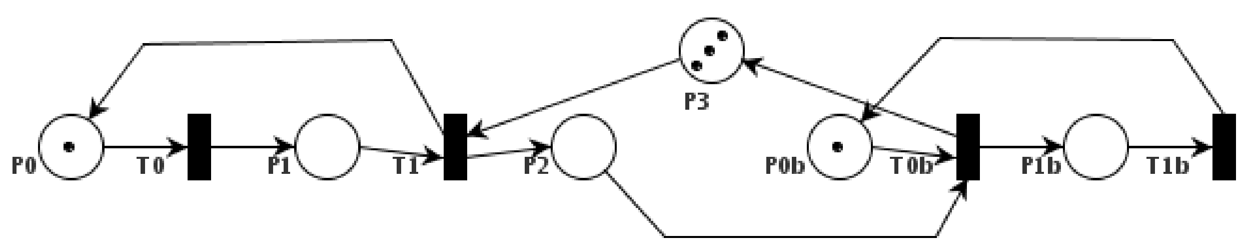
Boundedness and Saveness / Example (2/2)

The former example is not bounded as the first robot could produce arbitrary many tokens in P2 without the second robot ever consuming one.

Introducing a new place

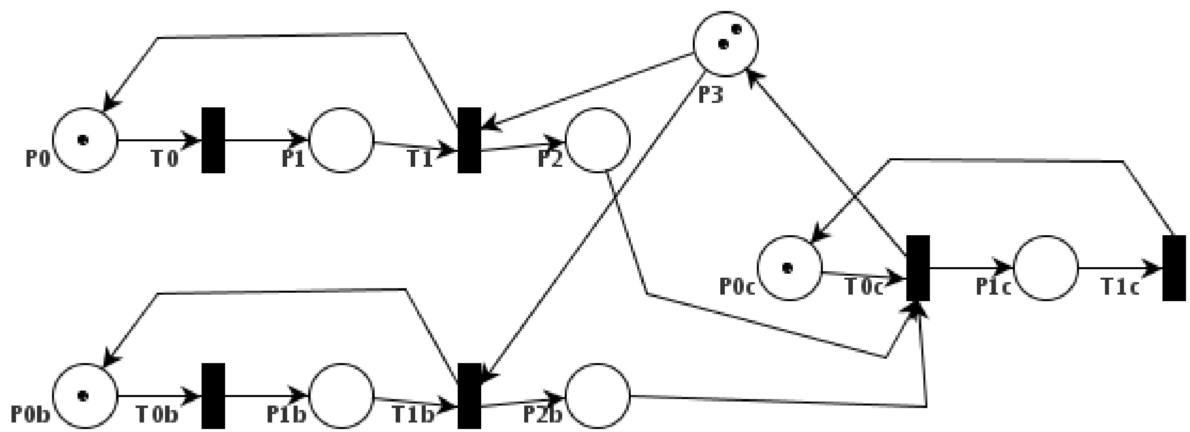
P3 buffer place available

with initially 3 tokens renders the example 3-bounded.



Deadlock

A mutex can easily produce a **deadlock**, i.e., all processes waiting for the availability of the mutex.



1. Petri Nets

2. The Pi Calculus

Overview

– The π-calculus is a another model for concurrent computation.

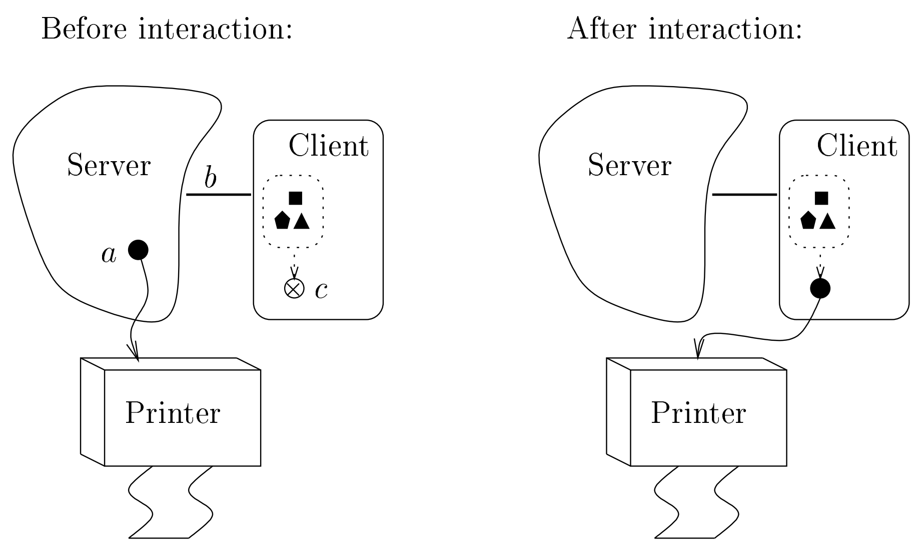
– The π-calculus is a formal language for defining concurrent communicating processes (usually called agents).

– The π-calculus relies on message passing between concurrent processes.

– The π-calculus got his name to resemble the lambda calculus, the minimal model for functional programming (Church/Kleene 1930s).
Here π (= greek p) as “parallel”.

– The π-calculus was invented by the Scottish mathematician Robin Milner in the 1990s.

Initial Example



[Par01]

$$(\bar{b}\langle a\rangle.S) \mid (b(x).\bar{x}\langle d\rangle.P) \xrightarrow{\tau} S \mid \bar{a}\langle d\rangle.P$$

Agents

Let $\mathcal{X}$ be a set of atomic elements, called **names**.

An **agent** is defined as follows:

$R ::=$	0	do nothing
	$\bar{x}\langle y\rangle.P$	send data y to channel x, then proceed as P
	$x(y).P$	receive data into y from channel x, then proceed as P
	$P + Q$	proceed either as P or as Q
	$P \mid Q$	proceed as P and as Q in parallel
	$(\nu x)P$	create fresh local name x
	$!P$	arbitrary replication of P, i.e., $P \vert P \vert P \vert \ldots$

where P, Q are agents and $x, y \in \mathcal{X}$ are names.

To modularize complex agents, one usually allows definitions of abbreviations as

$$A(x_1, x_2, \ldots, x_n) := P$$

as well as using such definitions

$$A(x_1, x_2, \ldots, x_n) \text{ proceed as defined by } A$$

where P is an agent and A is a name

Bound and Free Names

There are two ways to bind a name y in π-calculus:

– by receiving into a name: $x(y).P$.
– by creating a name: $(\nu y)P$.

All free / unbound names figure as named constants that agents must agree on:

agent	free names
0	$\emptyset$
$\bar{x}\langle y \rangle.P$	free$(P) \cup \{x, y\}$
$x(y).P$	free$(P) \setminus \{y\} \cup \{x\}$
$P + Q$	free$(P) \cup$ free(Q)
$P \mid Q$	free$(P) \cup$ free(Q)
$(\nu x)P$	free$(P) \setminus \{x\}$
$!P$	free(P)

Structural Congruence / Example

The same agent can be expressed by different formulas:

$$(\bar{b}\langle a \rangle.S) \,|\, (b(x).\bar{x}\langle d \rangle.P)$$

$$(b(x).\bar{x}\langle d \rangle.P) \,|\, (\bar{b}\langle a \rangle.S)$$

$$(b(y).\bar{y}\langle d \rangle.P) \,|\, (\bar{b}\langle a \rangle.S)$$

Therefore one defines a notion of equivalent formulas (structurally equivalent).

Structural Congruence

The following agents are said to be **structurally congruent**:

P	$\equiv Q$	if P and Q differ only in bound names
$P + Q$	$\equiv Q + P$	+-symmetry
$P + 0$	$\equiv P$	+-neutrality of 0
$P \mid Q$	$\equiv Q \mid P$	|-symmetry
$P \mid 0$	$\equiv P$	|-neutrality of 0
$!P$	$\equiv P \mid !P$	!-expansion
$(\nu x)0$	$\equiv 0$	restriction of null
$(\nu x)(\nu y)P$	$\equiv (\nu y)(\nu x)P$	ν-communtativity
$(\nu x)(P \mid Q)$	$\equiv P \mid (\nu x)Q$	if $x \notin$ free(P)

Formulas

$$(a(x).0) \quad \text{and} \quad (\bar{a}\langle b\rangle.0)$$

are abbreviated as

$$a(x) \quad \text{and} \quad \bar{a}\langle b\rangle$$

respectively.

Reduction

A **reduction** $P \to Q$ describes that P results in Q by parallel computation.

Reduction rules:

communication:

$$(\ldots + \bar{x}\langle z\rangle.P) \,|\, (\ldots + x(y).Q) \longrightarrow P \,|\, Q[z/y]$$

reduction under composition:

$$\frac{P \longrightarrow Q}{P \,|\, R \longrightarrow Q \,|\, R}$$

reduction under restriction:

$$\frac{P \longrightarrow Q}{(\nu x)P \longrightarrow (\nu x)Q}$$

same reduction for structurally equivalent agents:

$$\frac{P \longrightarrow Q \quad P \equiv P' \quad Q \equiv Q'}{P' \longrightarrow Q'}$$

Structured Messages

Often one agent needs to pass a message that consists of several parts.

Just sending both parts sequentially, may lead to garbled messages. Example:

$$(a(x).a(y)) \,|\, (\bar{a}\langle b_1\rangle.\bar{a}\langle c_1\rangle) \,|\, (\bar{a}\langle b_2\rangle.\bar{a}\langle c_2\rangle)$$

intends to sent either (b_1, c_1) or (b_2, c_2) and bind it to (x, y), but it may happen that actually the second agent sents b_1, then the thrird b_2, so (x, y) is bound to (b_1, b_2).

Private channels can avoid this problem:

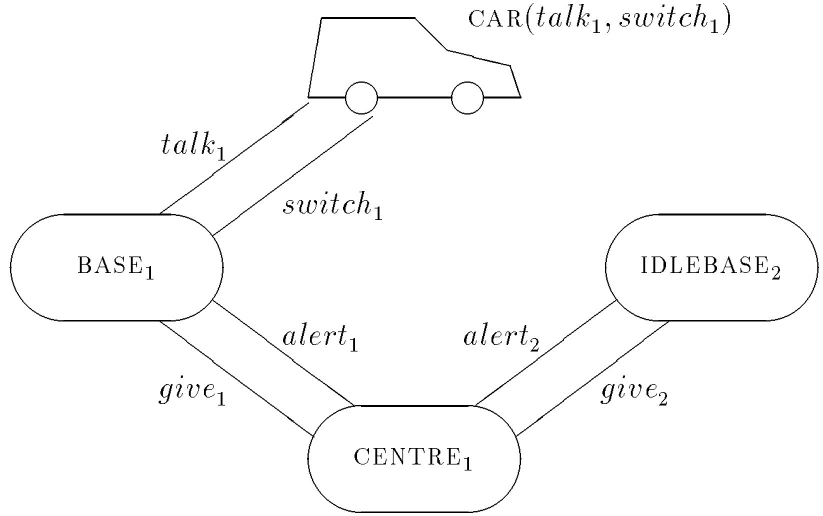
$$\bar{a}\langle b_1, b_2, \cdots, b_n\rangle := (\nu w)(\bar{a}\langle w\rangle.\bar{w}\langle b_1\rangle.\bar{w}\langle b_2\rangle.\cdots.\bar{w}\langle b_n\rangle)$$
$$a(x_1, x_2, \cdots, x_n) := a(w).w(b_1).w(b_2).\cdots.w(b_n)$$

Now the example can we written as

$$a(x, y) \,|\, \bar{a}\langle b_1, c_1\rangle \,|\, \bar{a}\langle b_2, c_2\rangle$$

and just the private channel name w is exchanged via the public channel a,
the actual data (b_1, c_1) is sent via the private channel w.

An Example (1/3)



[Mil93]

4 concurrent agents: car, two bases and centre.

8 named channels: talk t_1, t_2, switch s_1, s_2, give g_1, g_2, alert a_1, a_2.

First base uses channels t_1 and s_1 to communicate with car, g_1 and a_1 to communicate with centre.

Second base uses channels t_2 and s_2 to communicate with car, g_2 and a_2 to communicate with centre.

An Example (2/3)

$$\begin{aligned}
\text{System}_1 :=& (\nu t_1, t_2, s_1, s_2, g_1, g_2, a_1, a_2) \\
& (\text{Car}(t_1, s_1) \,|\, \text{Base}(t_1, s_1, g_1, a_1) \,|\, \text{IdleBase}(t_2, s_2, g_2, a_2) \,|\, \text{Centre}_1) \\
\text{Car}(t, s) :=& t().\text{Car}(t, s) + s(t', s').\text{Car}(t', s') \\
\text{Base}(t, s, g, a) :=& t().\text{Base}(t, s, g, a) + g(t', s').\bar{s}\langle t', s' \rangle.\text{IdleBase}(t, s, g, a) \\
\text{IdleBase}(t, s, g, a) :=& a().\text{Base}(t, s, g, a) \\
\text{Centre}_1 :=& \bar{g}_1\langle t_2, s_2 \rangle.\bar{a}_2\langle\rangle.\text{Centre}_2 \\
\text{Centre}_2 :=& \bar{g}_2\langle t_1, s_1 \rangle.\bar{a}_1\langle\rangle.\text{Centre}_1
\end{aligned}$$

An Example (3/3)

$$\text{System}_1 := (\nu t_1, t_2, s_1, s_2, g_1, g_2, a_1, a_2)$$
$$(\text{Car}(t_1, s_1) \,|\, \text{Base}(t_1, s_1, g_1, a_1) \,|\, \text{IdleBase}(t_2, s_2, g_2, a_2) \,|\, \text{Centre}_1)$$
$$\equiv \ldots \,|\, (\ldots + g_1(t', s').\bar{s}_1\langle t', s'\rangle.\text{IdleBase}(t_1, s_1, g_1, a_1) \,|\, \ldots \,|\, (\bar{g}_1\langle t_2, s_2\rangle.\bar{a}_2\langle\rangle.\text{Centre}_2)$$

$$\rightarrow \ldots \,|\, (\bar{s}_1\langle t_2, s_2\rangle.\text{IdleBase}(t_1, s_1, g_1, a_1) \,|\, \ldots \,|\, (\bar{a}_2\langle\rangle.\text{Centre}_2)$$
$$\equiv (\ldots + s_1(t', s').\text{Car}(t', s')) \,|\, (\bar{s}_1\langle t_2, s_2\rangle.\text{IdleBase}(t_1, s_1, g_1, a_1) \,|\, \ldots \,|\, (\bar{a}_2\langle\rangle.\text{Centre}_2)$$

$$\rightarrow \text{Car}(t_2, s_2) \,|\, \text{IdleBase}(t_1, s_1, g_1, a_1) \,|\, \ldots \,|\, (\bar{a}_2\langle\rangle.\text{Centre}_2)$$
$$\equiv \text{Car}(t_2, s_2) \,|\, \text{IdleBase}(t_1, s_1, g_1, a_1) \,|\, (a_2().\text{Base}(t_2, s_2, g_2, a_2)) \,|\, (\bar{a}_2\langle\rangle.\text{Centre}_2)$$

$$\rightarrow \text{Car}(t_2, s_2) \,|\, \text{IdleBase}(t_1, s_1, g_1, a_1) \,|\, \text{Base}(t_2, s_2, g_2, a_2) \,|\, \text{Centre}_2$$

References

[Mil93] Robin Milner. The polyadic pi-calculus: A tutorial. In F. L. Hamer, W. Brauer, and H. Schwichtenberg, editors, *Logic and Algebra of Specification*. Springer, 1993.

[Par01] Joachim Parrow. An introduction to the π-calculus. In Jan A. Bergstra, Alban Ponse, and Scott A. Smolka, editors, *Handbook of Process Algebra*. Elsevier, 2001.