Realistic Optimal Policies For Energy-Efficient Train Driving

Josif Grabocka, Alexandros Dalkalitsis, Athanasios Lois, Evangelos Katsaros and Lars Schmidt-Thieme

***Abstract*— Transportation is a crucial cog within the cogwheel of our economies and modern lifestyles. Unfortunately, both the rising cost of energy production and the increasing demand for transportation pose the challenge of minimizing the energy consumption of automobiles. This paper proposes an offline driver behavior adaptation approach (*eco-driving*) for trains. An optimal driving behavior policy is computed using Simulated Annealing optimization search over a collection of real driving behavior data (realistic policy). Empirical findings show that if drivers would follow the recommended optimal policy, then an energy saving of up to 50 % is a realistic upper bound potential.**

I. INTRODUCTION

Transportation is a backbone of modern economies that catalyzes the connectivity of businesses and the motion of supplies across long distances. The unfavorable rising cost of energy production has emphasized the importance of optimizing the energy consumption of transportation vehicles. In that prism, nations are striving to maintain a win-win balance, which copes with the increasing need for transportation, while at same time avoids additive figures on their cumulative energy bills. Population and economy growths make it hard (if not impossible) to reduce the total number of travels and deliveries, therefore solutions need to be found in order to reduce the energy consumption by preserving the total traffic throughput. One could categorize **at least** three key approaches for optimizing energy consumption: (i) technological innovations (more efficient engines, aerodynamics, tires, etc.), (ii) infrastructure and administrative regulations (shorter roads, speed limits, etc.) [14], and (iii) computer science methodologies (algorithms for ecological routes, driving behavior recommendation systems, etc.) [18]. This paper lies within the scope of the (iii)-rd approach and proposes a novel method that helps optimizing the energy consumption through an algorithm that recommends the adaptation of ecological driving behaviors.

More specifically the process of adapting one's driving behaviors, by suggesting energy-efficient driving patterns, is also known as either *energy-efficient driving* or *eco-driving recommendation* [1]. Eco-driving can be conducted online during the travel [9], or offline in the form of a training course [1]. Several eco-driving aspects have been elaborated in the literature, with a concise list presented in Section II.

In contrast to existing work that largely concentrates on tire-based vehicles (cars, trucks, buses, etc,...), this paper introduces an offline eco-driving method for **trains**. Furthermore, this paper addresses the problem of computing the *optimal driving behavior* between two arbitrary destinations.

Since the primary objective of this work is to save energy, we define *optimal* driving as being **the** driving behavior which results in the universally minimum energy consumption, by respecting travel time constraint. Unfortunately, the existence of several real-life factors which cannot be measured or taken into analysis make it very hard to compute universally optimal policies through mathematical analysis. Fortunately, the availability of measurements (big data paradigm) that record driving data of train trips offers a possibility to tackle the problem alternatively using a data mining perspective.

Before providing a highlight of our method, we would like to clarify one term. By optimal policy we identify a velocity series with respect to the relative distance from the start of the travel. In other words, an optimal policy answer the following question: "While driving between arbitrary departure and arrival station, what is the best velocity/speed a train driver should follow at every distance point (e.g. at each kilometer stone)?".

A collection of driving data from the train fleet of TrainOSE S.A., Greece is utilized. The data consists of multivariate time-series composed of time, relative distance, speed and instantaneous energy variables. For instance, Figure 1 shows the velocity vs distance series corresponding to travels conducted by different train drivers between the Domokos and Thessaloniki stations in Greece. In addition, there is one energy series associated to every speed series, which we denote as the velocity-energy bi-variate series.

The proposed method mines through all the velocity series and the instantaneous energy series, in order to compute the best driving policy, i.e. the series that has the minimum energy consumption. We will demonstrate that the optimal policy can be constructed by concatenating low-consumption sub-sequences of different velocity-energy series. The search for the optimal policy is carried using a non-convex optimization method named Simulated Annealing. The search starts with the velocity time series which has the minimum recorded total energy and improves it by perturbating local segments. Because the segments of the optimal policy are extracted from driving data series of real drivers, then the end result is a realistic (practically feasible) policy. We applied the proposed method to real data and the derived optimal policies demonstrate a potential for up to 50% reduction of the used energy.

Josif Grabocka and Lars Schmidt-Thieme are with Information Systems and Machine Learning Lab, University of Hildesheim, Germany, {josif, schmidt-thieme}@ismll.uni-hildesheim.de

Alexandros Dalkalitsis, Athanasios Lois and Evangelos Katsaros are with TrainOSE S.A., Greece {a.dalkalitsis, lois, e.katsaros}@trainose.gr

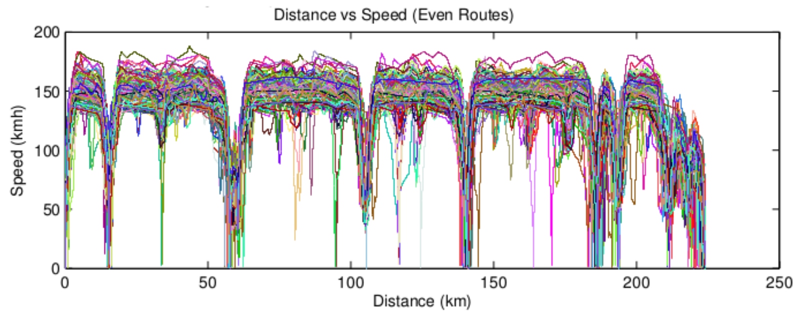

Fig. 1. 385 velocity series for the trips between the Domokos and Thessaloniki stations (Even numbered routes (50,52,. . . , 60)).

II. RELATED WORK

Given the economical importance of saving energy, a large number of researchers have dedicated efforts towards proposing eco-driving related methodologies. The purpose of this section is to mention only a fraction of highly related works, instead of exhaustively enumerating the existing work.

Eco-driving for motorway automobiles is a vastly researched topic. In particular, the adaptation of driving skills has been a focus of energy optimization analysis [18]. Communication technologies, on the other hand, have been utilized to provide online eco-driving assistance using either Vehicle-to-Infrastructure communication [15], or distributed Vehicle-to-Vehicle infrastructure [2]. A more analytic approach has elaborated the identification of the most influential factors over fuel consumption [12].

Online driving assistance have been tailored to consider contextualized recommendation using CANBUS data [9]. Furthermore, research efforts have been spent to devise online systems that continuously encourage efficient driving behavior [17]. In addition, online driving assistance have been applied in hilly roads with up-down slopes [10].

Fuel Economy Optimization Systems typically involve computations of optimal velocity and acceleration [20], [11]. Another study extends optimal control frameworks of driving assistance systems for predicting the dynamics of other vehicles [19]. From another perspective, optimal policies have been computed for identifying the safest driving patterns [7].

In terms of eco-driving for trains, the computation of reference trajectories have been conducted for automatic train operation [8]. Speed profiles have been calculated from simulation models in order to derive energy-efficient driving strategies [13]. Last but not least, the optimization of driving speed of trains has been further proven beneficial in reducing the overall traction energy [5], [6].

In comparison to the existing literature, we introduce a novel method that computes the optimal policy for train driving. Our approach explores realistic policies that are extracted through a large amount of driving behavior data from different drivers.

III. SEARCHING THE OPTIMAL POLICY

A. Problem Definition

The input data consists of N-many bi-variate velocity-energy series, each having a length of L-points. The velocity series are denoted as $V \in \mathbb{R}^{N \times L}$, while the energies as $E \in \mathbb{R}^{N \times L}$. We would like to stress that energy measures instantaneous consumption. The consumption at distance point l is denoted as E_l^{opt} and records how much energy was spent since the measurement at the $l-1$ distance point. The final definition of the problem is formalized in Equation 1. Semantically we search for the optimal policy $V^{\text{opt}} \in \mathbb{R}^L$ that results in the minimal energy $E^{\text{opt}} \in \mathbb{R}^L$.

$$\left(V^{\text{opt}}, E^{\text{opt}}\right) = \underset{\left(V', E'\right)}{\text{argmin}} \sum_{l=1}^{L} E_l^{'} \tag{1}$$

$$\forall l \in \mathbb{N}_1^L : \exists n \in \mathbb{N}_1^N \text{ s.t. } V_l^{'} = V_l^{(n)}, E_l^{'} = E_l^{(n)} \tag{2}$$

The objective function of Equation 1 is subject to an important constraint that is emphasized in Equation 2. The constraint ensures that every point l of the optimal velocity and energy series $V^{\text{opt}}, E^{\text{opt}}$ must belong to the l-th point of some series bi-variate series (v, e) from our input data (V, E). Therefore, we ensure that the optimal policy is realistic, in terms of occurring in real data series.

B. Principle

Naturally the first intuition towards solving the problem of this paper needs to answer a trivial question: "Why can't we use the series having the minimal energy as our optimal policy?". Whilst the minimum recorded energy series is a candidate, one can further optimize that policy by incorporating segments of other series.

Figure 2 provides an example that motivates the principle behind our method. The upper plot includes two velocity series in red and blue, marked as E1 and E2, each having different total energies. In this synthetic dataset the red series E2 has the minimal recorded energy of 900 kwh, while the blue series E1 has a worse total energy consumption of 1000 kwh. Yet there is one segment delimited by vertical dashed lines where the blue series is more efficient that the red series. Concretely, the blue line has an energy consumption of 300 kwh for the sub-sequence, that is smaller than the 350 kwh consumed in the red series. Such an illustration shows that a cumulatively energy-minimal series can still have non-efficient sub-sequences.

The optimal policy, on the other hand, can be simply derived by concatenating the most energy-efficient sub-

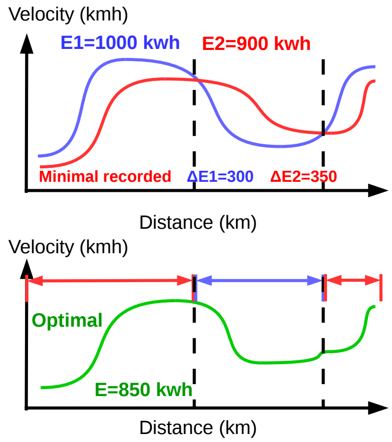

Fig. 2. Improving the driving policy (velocity vs. distance series) by swapping segments

sequences/segments of different series, as shown in the lower plot of Figure 2. The optimal policy shown in green color can be constructed by taking the recorded minimal E1 and replacing its content between the dashed vertical lines with the content of the blue E2 series. In that way, the overall consumption of the optimal policy is reduced to $900 - 350 + 300 = 850$ kwh.

C. Perturbation of a Velocity-Energy Bi-Variate Series

This section simply formalizes the procedure of swapping the contents of segments between two series, which we denote as *perturbation*. In the forthcoming search algorithm, the perturbation of the series having minimal recorded energy will provided useful candidates toward finding more energy-efficient policies.

Algorithm 1 describes the pseudo-code needed to generate a *random* perturbation of a current input bi-variate series, denoted as $V^{\text{curr}}, E^{\text{curr}}$. The perturbation of the current velocity-energy bi-variate series is conducted by first selecting two random points $p1, p2$ in lines 2-3 of the algorithm. Next, a search is conducted in lines 4-5 to check whether there is any series V_i from our data V, where the velocity endpoint values of the segments match.

It is important to realize that we can not simply swap segment content that create non-smooth policies with disconnected segments. Yet, we tolerate the endpoint segments values from the two different series to be within an ϵ distance (in our setting $\epsilon = 3$ kmh). In case there do exists a series where the velocity values are close at the segment endpoints p_1, p_2, then the swapping step is executed in lines 6-7. Both the new velocity and energy series, denoted $V^{\text{next}}, E^{\text{next}}$, are created by concatenating existing parts of the input series $V^{\text{curr}}, E^{\text{curr}}$ and the segment from series V_i, E_i. After the perturbation the loop breaks (algorithm returns) in line 8. Otherwise, if no matching series V_i is found, then two other random segment endpoints are selected and the algorithm is repeated until a perturbation is created.

Algorithm 1: Perturbate a Policy Using Real Data

Data: Current Policy Speed: $V^{\text{curr}} \in \mathbb{R}^L$,Current Policy Energy: $E^{\text{curr}} \in \mathbb{R}^L$, Velocity series: $V \in \mathbb{R}^{N\times L}$, Energy series: $E \in \mathbb{R}^{N\times L}$, Swapping threshold $\epsilon \in \mathbb{R}$
Result: Perturbated policy: $V^{\text{next}} \in \mathbb{R}^L$, $E^{\text{next}} \in \mathbb{R}^L$

1 **while** ***true*** **do**
2   Draw a random point $p_1 \in \mathcal{U}(1,\dots,L)$;
3   Draw a random point $p_2 \in \mathcal{U}(p_1+1,\dots,L)$;
4   **if** $\exists V_i \mid i \in [1,\dots,N] \wedge \left(V_{i,p1} - V^{curr}_{p1} < \epsilon\right)$
5         $\wedge \left(V^{curr}_{p2} - V_{i,p2} > \epsilon\right)$ **then**
6         $V^{\text{next}} \leftarrow [V^{\text{curr}}_{1:p1-1} \oplus V_{i,p1:p2} \oplus V^{\text{curr}}_{p2+1:L}]$;
7         $E^{\text{next}} \leftarrow [E^{\text{curr}}_{1:p1-1} \oplus E_{i,p1:p2} \oplus E^{\text{curr}}_{p2+1:L}]$;
8   **break**;
9   **end**
10 **end**
11 **return** V^{next}, E^{next}

D. Simulated Annealing Search

The minimal recorded series is a good start towards discovering the optimal policy. Still further improvements can be achieved through swapping the energy-inefficient segments of the optimal policy with more energy-efficient segments of other bi-variate series.

We are going to use a search method called *Simulated Annealing* [16] to conduct the search for the optimal policy. The idea of the search is to start with the minimum as a current solution and generate lots of candidates through perturbation. The Simulated Annealing is presented with the means of Algorithm 2.

Algorithm 2: Simulated Annealing: Finding The Optimal Policy

Data: Velocity series $V \in \mathbb{R}^{N\times L}$, Energy series $E \in \mathbb{R}^{N\times L}$
Result: Optimal policy $V^{\text{opt}} \in \mathbb{R}^L$

1 $[V^{\text{opt}}, E^{\text{opt}}] \leftarrow [V^{\text{curr}}, E^{\text{curr}}] \leftarrow$
  $\{[V_i, E_i] \mid \neg\exists\, j \;\text{ s.t. } \sum E_j < \sum E_i,\; i,j \in [1,\dots,N]\}$;
2 **for** $\mathcal{T} = \mathcal{T}_{max},\dots,\mathcal{T}_{min}$ **do**
3     $[V^{\text{next}}, E^{\text{next}}] \leftarrow \text{Perturbate}(V^{\text{curr}}, E^{\text{curr}}, V, E)$;
4     $\Delta E = \sum E^{\text{next}} - \sum E^{\text{curr}}$;
5   **if** $\Delta E < 0 \vee rand(0,1) < e^{-\frac{\Delta E}{\mathcal{T}}}$ **then**
6         $[V^{\text{curr}}, E^{\text{curr}}] \leftarrow [V^{\text{next}}, E^{\text{next}}]$;
7   **if** $\sum E^{curr} < \sum E^{opt}$ **then**
8             $[V^{\text{opt}}, E^{\text{opt}}] \leftarrow [V^{\text{curr}}, E^{\text{curr}}]$;
9   **end**
10   **end**
11 **end**
12 **return** V^{opt}

The algorithm starts by initializing two velocity-energy bi-variate series in line 1, denoted by the superscripts current and optimal. The current series identifies the latest perturbation in a sequential (iterative) style, while the optimal series record the series with the minimal total energy yielded so far. During each iteration (line 2), the next series (line 3) can have a smaller or larger total energy. We record the difference in total energy consumption before and after the perturbation as ΔE in line 4. If the next series has a lower energy ($\Delta E < 0$) then it is accepted directly in line 5 and the perturbated next series is copied to the current solution (line 6), in order to start a new perturbation. During each next accepted solution we check whether the new current series has the minimum/optimal so far energy (lines 7-9).

An important aspect of Simulated Annealing is the conditional acceptance of perturbated series that worsen energy consumption. In order to avoid local optima, one should allow occasional divergence steps during the search. The iteration variable, denoted as $\mathcal{T}$, represent metaphorically the search temperature. The conditional acceptance is carried only if a random number between 0 and 1 is smaller than $e^{\frac{\Delta E}{\mathcal{T}}}$. Such a temperature metaphor is inherited from the field of metallurgy where metals are heated to high temperature where random perturbation of the molecular structure are allowed. Gradually the temperature of the metal is cooled down while the molecules arrange themselves towards a minimum bond energy. Further discussions on the mechanism of Simulated Annealing can be found in [16].

E. Convergence of The Search

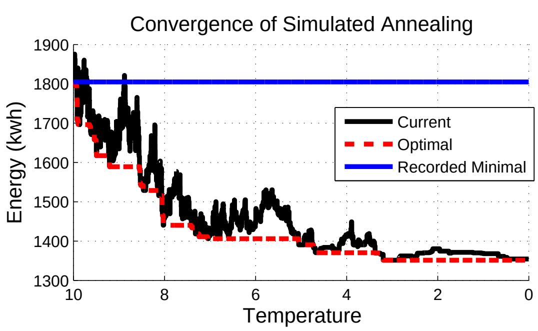

Fig. 3. Illustration of Simulated Annealing convergence using trip series whose total travel times are between 100 and 118 minutes.

The convergence of the Simulated Annealing terminates when the temperature $\mathcal{T}$ approaches the specified minimum temperature $\mathcal{T}_{min}$. Figure 3 illustrates the convergence of the search algorithm. Initially, both the current series (shown in black) and the optimal policy series (shown in red) are initialized to the minimal recorded series (shown in blue). As next perturbations are generated the current line goes down ($\Delta E < 0$), or oscillates up for conditional probabilistic acceptances.

Overall the current series keeps resulting in smaller and smaller total energies as the temperature (x-axis) cools down. While approaching the minimum energy ($\mathcal{T} \rightarrow \mathcal{T}_{min}$) there are less oscillations because $e^{\frac{\Delta E}{\mathcal{T}}}$ approaches zero. Such a stability in terms of the changes of the current and the optimal policies is known as the convergence of the search algorithm.

IV. EXPERIMENTAL RESULTS

A. Travel Time Constraint

Before presenting the results of the optimal policies, it is important to clarify a constraint on the travel time. An energy-efficient style of driving typically involve slower velocities. However, a train also needs to arrive in time at the destination. In order to ensure such a constraint we will divide the data into three groups based on the histogram of travel times, shown in Figure 5.

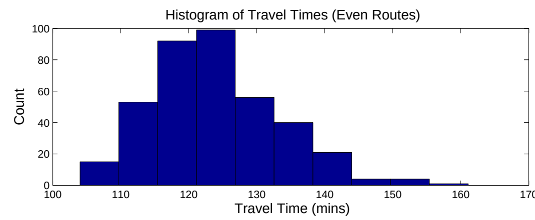

Fig. 5. Histogram of the travel times of all travels between the Domokos and Thessaloniki stations

The trips are divided into three groups by dividing the histogram into three equi-volume chunks, as follows:

- Travel time between $[100, 118)$ minutes: Fast trips
- Travel time between $[118, 130)$ minutes: Normal trips
- Travel time between $[130, 160]$ minutes: Slow trips

B. Optimal Policies Per Travel Time

The optimal policies for all the three travel time segments are shown in Figure 4. The blue line denotes the recorded minimum, while the red line the optimal policy achieved by the search algorithm. The used search parameters are $\mathcal{T}_{max} = 10, \mathcal{T}_{min} = 0$ with $\mathcal{T}$ decremented by 0.0003.

Regarding the slow trips (time $[100, 118)$ mins), we observe that the optimal policy results in a total consumption of 1236.7 kwh, compared to the 1805 kwh of the minimal recorded. Please note that the saving potential of the optimal policy is 31% better than the best recorded policy. An inspection of the optimal policy reveals that the optimal policy enforces lower velocity values and naturally lower energy consumption.

The improvement of the optimal policy is also visible in both remaining travel time segments ($[118, 130)$ mins, $[130, 160]$ mins). In the case of normal trips, the optimal energy results in a total consumption of 1002.6 kwh. That is 104.4 kwh better than the recorded minimal policy. Similarly, the optimal policy has 493.6 kwh less consumption than the minimal policy.

C. Analysis of Potential Savings

It is of important value to assess the potential economical impact that our optimal policy might achieve. Under such a scope, we are presenting the amount of money a train fleet company could save if an average driver would strictly follow our suggested optimal policy. The *per-trip* average and the minimal and optimal figures are displayed in Table I for three

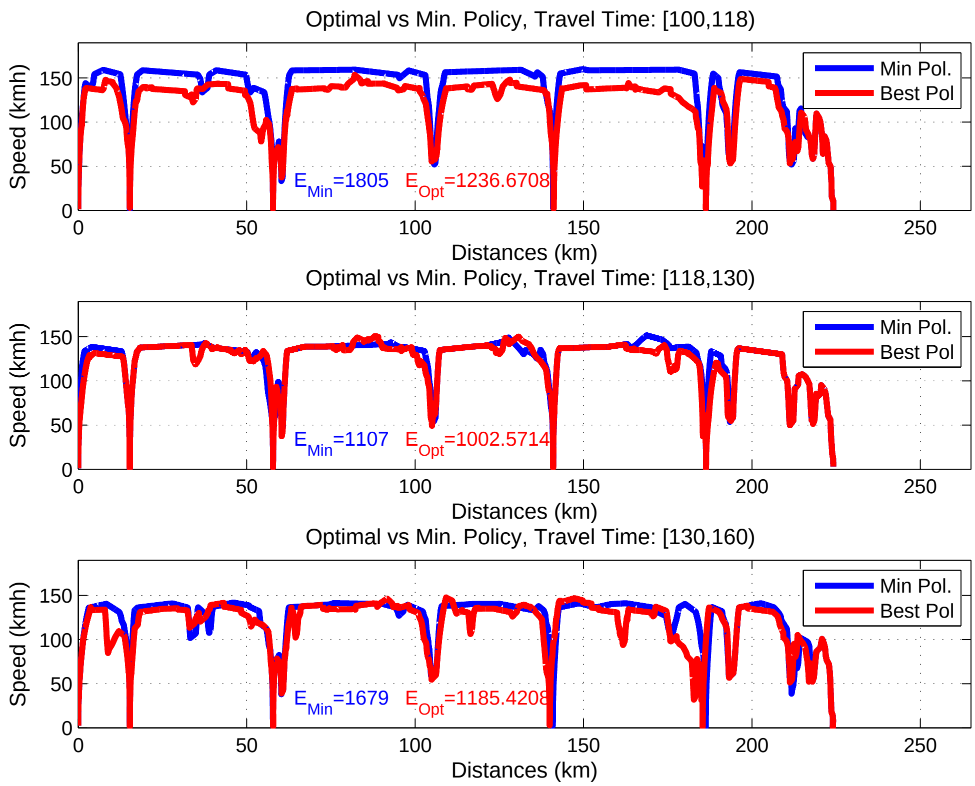

Fig. 4. Optimal Train Driving Policies; Time units in *mins*, Energy units in *kwh*.

criteria: energy consumption, monetary cost and the green house gas emission. We would like to note that the figures were round up for the sake of presentability.

	Time (min)	**Average**	**Minimal**	**Optimal**
Energy (kwh)	100-118	2290	1800	1240
	118-130		1100	1000
	130-160		1680	1190
Cost (euro)	100-118	254	200	140
	118-130		125	110
	130-160		190	130
CO2 (kg)	100-118	2244	1170	1210
	118-130		1080	980
	130-160		1645	1160

TABLE I

ENERGY CONSUMPTION (KWH), MONETARY COST (EURO) AND THE GREEN HOUSE GAS FOOTPRINT (KG OF CO_2)

The energy is measured directly from the locomotives from every trip, while the average figure is presented for all the trips between the Domokos and Thessaloniki stations. On the other hand, the monetary cost is derived from EuroStat's electricity price per kwh energy [4]. The CO_2 emission is computed from the energy figures using the EcoTransIT initiative's convertion rates [3].

Therefore, the potential energy, cost and emission reductions which could be achieved if an average driver would follow the optimal policy are up to: 45% for time [100,118) mins, 56% for time [118,130) mins and 48% for time [130,160] mins. Since our optimal policy is constructed using real driving data, we can deduce that a reduction of approximately 50% is a realistic upper bound potential.

V. CONCLUSION

The optimization of energy consumption is a key trend in transportation research. This paper proposed a novel method that computes an off-line optimal driving policy for trains. Our method operates over real train driving data, ensuring this way that the derived optimal policy is realistically feasible. Simulated Annealing is used to discover the optimal velocity policy by starting from the minimal recorded series and optimizing it through swapping segments with other recorded series. By perturbating (swapping) arbitrary segments of a current velocity-energy bi-variate series, a lot of further candidates are created. The candidates are conditionally accepted by evaluating whether they reduce the total energy consumption, or not. Finally, the results of the optimal policies were presented for three trip categories, grouped based on their travel times into fast, normal and slow trips. The optimal policy discovered using our novel method improves the energy consumption of the best (minimum) recorded series by up to 31%. In case an average driver would follow our suggested optimal (yet realistic) velocity policy, the overall consumption could potentially be reduced to approximately 50%. Economically speaking the potential monetary saving, as well as the reduction in Green House Gas emission is also up to 50%.

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